1. Find the indicated limit.

   (a) $\lim_{x\to 0} \frac{\sin(2x)}{3x - 2x^3}$

$$\lim_{x\to 0} \frac{\sin(2x)}{3x-2x^3} \overset{\checkmark}{=} \lim_{x\to 0} \frac{2\cos(2x)}{3-6x^2} = \frac{2}{3}$$

   (b) $\lim_{x\to 0} \frac{e^{(x^2+3x)} - 1}{x+1}$

$$\lim_{x\to 0} \frac{e^{(x^2+3x)}-1}{x+1} \overset{\checkmark}{=} \lim_{x\to 0} \frac{(2x+3)\, e^{(x^2+3x)}}{1} = 3$$

2. Let $a_n = \frac{7n}{9n-1}$, $n \geq 1$.

   (a) Does this sequence converge or diverge? If it converges, state the limit.

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{7n}{9n-1} \overset{\checkmark}{=} \frac{7}{9}$$

The sequence converges to $\frac{7}{9}$.

   (b) Does $\sum_{n=1}^{n=\infty} a_n$ converge or diverge? If it converges, state the sum.

Since the sequence does not converge to 0,
the series diverges by the Test for Divergence.

3. Determine if the series $\sum_{n=1}^{n=\infty} \frac{5^n}{3^n+2}$ converges or diverges.

$$\lim_{n\to\infty} \frac{5^n}{3^n+2} \overset{\checkmark}{=} \lim_{n\to\infty} \frac{5^n \ln 5}{3^n \ln 3} \overset{\checkmark}{=} \frac{\ln 5}{\ln 3} \lim_{n\to\infty} \left(\frac{5}{3}\right)^n = \infty.$$

The series diverges by the Test for Divergence.

4. Determine if the series $\sum_{n=1}^{n=\infty} \frac{(-1)^{n+1}}{n+4}$ is absolutely convergent, conditionally convergent, or divergent.

Alternating Series Test :

1) $\lim_{n\to\infty} \frac{1}{n+4} = 0$ $\checkmark$

2) $\frac{1}{n+5} \leq \frac{1}{n+4}$ $\checkmark$

So the series is convergent, but which kind?

The absolute value series is $\displaystyle\sum_{n=1}^{n=\infty} \frac{1}{n+4}$

This is similar to the series $\displaystyle\sum_{n=1}^{n=\infty} \frac{1}{n}$, which diverges (Harmonic Series).

$$\lim_{n\to\infty} \frac{1/n}{1/(n+4)} = \lim_{n\to\infty} \frac{n+4}{n} \overset{\checkmark}{=} 1$$

So $\displaystyle\sum_{n=1}^{n=\infty} \frac{1}{n+4}$ diverges by the Limit Comparison Test.

$\displaystyle\sum_{n=1}^{n=\infty} \frac{(-1)^{n+1}}{n+4}$ is conditionally convergent.

5. Find the radius of convergence of $\displaystyle\sum_{n=1}^{n=\infty} \frac{(2x-3)^n}{n^2 8^n}$.

$$\begin{aligned}
\lim_{n\to\infty} \left| \frac{\left(\dfrac{(2x-3)^{n+1}}{(n+1)^2\, 8^{n+1}}\right)}{\left(\dfrac{(2x-3)^n}{n^2 8^n}\right)} \right| &= \lim_{n\to\infty} \left|\frac{2x-3}{8}\right| \frac{n^2}{(n+1)^2} \\
&= \left|\frac{2x-3}{8}\right| \lim_{n\to\infty} \frac{n^2}{(n+1)^2} \\
&\overset{\checkmark}{=} \left|\frac{2x-3}{8}\right| \lim_{n\to\infty} \frac{2n}{2n+2} \\
&\overset{\checkmark}{=} \left|\frac{2x-3}{8}\right|
\end{aligned}$$

$$\begin{aligned}
\left|\frac{2x-3}{8}\right| &< 1 \\
-1 &< \frac{2x-3}{8} < 1 \\
-8 &< 2x-3 < 8 \\
-5 &< 2x < 11 \\
-\frac{5}{2} &< x < \frac{11}{2}
\end{aligned}$$

The radius of convergence is $\dfrac{\dfrac{11}{2} - \left(-\dfrac{5}{2}\right)}{2} = 4$

6. Short Answer.

(a) Make a careful sketch of the function $f(x) = 6^{x-3} + 2$.

This is the graph of $f(x) = 6^x$ shifted 3 to the right and 2 up.
The graph is an exponential passing through the points $(3,3)$ and $(4,8)$, with a horizontal asymptote of $y = 2$.

(b) Let $\{a_n\}_{n\geq 1}$ be given by $\left\{\frac{5}{3}, -\frac{7}{9}, \frac{9}{27}, -\frac{11}{81}, \frac{13}{243}, \ldots\right\}$. Find a formula for a_n.

$$a_n = (-1)^{n+1} \frac{2n+3}{3^n}$$

7. Short Answer.

(a) Does the interval of convergence of the series $\sum_{n=1}^{n=\infty} \frac{(5x-4)^n}{n^6}$ include the value $x = 1$?

$$\begin{aligned} \sum_{n=1}^{n=\infty} \frac{(5(1)-4)^n}{n^6} &= \sum_{n=1}^{n=\infty} \frac{(1)^n}{n^6} \\ &= \sum_{n=1}^{n=\infty} \frac{1}{n^6} \end{aligned}$$

This series converges ($p = 6$).
$x = 1$ is included in the interval of convergence.

(b) Find the exact value of $\sum_{n=1}^{\infty} \frac{3(-2)^n}{5^{n+2}}$.

$$\begin{aligned} \sum_{n=1}^{\infty} \frac{3(-2)^n}{5^{n+2}} &= \sum_{n=1}^{\infty} \frac{3}{25}\left(-\frac{2}{5}\right)^n \\ &= \frac{(3/25)(-2/5)}{1-(-2/5)} \\ &= \frac{-6/125}{7/5} \\ &= -\frac{6}{175} \end{aligned}$$

(c) Find the exact value of $\sum_{n=0}^{\infty} \frac{2}{n^3+1} - \sum_{n=3}^{\infty} \frac{2}{n^3+1}$.

$$
\begin{aligned}
\sum_{n=0}^{\infty} \frac{2}{n^3+1} - \sum_{n=3}^{\infty} \frac{2}{n^3+1} &= \frac{2}{0^3+1} + \frac{2}{1^3+1} + \frac{2}{2^3+1} \\
&= 2 + 1 + \frac{2}{9} \\
&= \frac{29}{9}
\end{aligned}
$$