1. Find $f'(x)$.

(a) $f(x) = \tan^{-1}(5x+1)$

$$\begin{aligned} f'(x) &= \frac{1}{1+(5x+1)^2}(5) \\ &= \frac{5}{1+(5x+1)^2} \end{aligned}$$

(b) $f(x) = (4x)^{\sin x}$

$$\begin{aligned} f(x) &= e^{\ln(4x)^{\sin x}} \\ &= e^{\sin x \ln 4x} \end{aligned}$$

$$\begin{aligned} f'(x) &= e^{\sin x \ln 4x}\left(\sin x\left(\frac{1}{4x}4\right) + \cos x \ln 4x\right) \\ &= (4x)^{\sin x}\left(\frac{\sin x}{x} + \cos x \ln 4x\right) \end{aligned}$$

2. Find the Taylor Series centered at $a = 2$ for the function $f(x) = e^{3x}$. In your answer either use sigma notation or show at least five non-zero terms in your series.

$$\begin{array}{lll} f(x) = e^{3x} & f(2) = e^6 & c_0 = e^6 \\ f'(x) = 3e^{3x} & f'(2) = 3e^6 & c_1 = 3e^6 \\ f''(x) = 9e^{3x} & f''(2) = 9e^6 & c_2 = \frac{9e^6}{2!} \\ f'''(x) = 27e^{3x} & f'''(2) = 27e^6 & c_3 = \frac{27e^6}{3!} \\ f^{(4)}(x) = 81e^{3x} & f^{(4)}(2) = 81e^6 & c_4 = \frac{81e^6}{4!} \\ \vdots & \vdots & \vdots \end{array}$$

$$e^{3x} = e^6 + 3e^6(x-2) + \frac{9e^6}{2!}(x-2)^2 + \frac{27e^6}{3!}(x-2)^3 + \frac{81e^6}{4!}(x-2)^4 + \ldots = \sum_{n=0}^{\infty} \frac{3^n e^6 (x-2)^n}{n!}$$

3. Evaluate $\int x^2 \cos(2x)\,dx$

	Diff	Anti-D
$+$	x^2	$\cos 2x$
$-$	$2x$	$\frac{1}{2}\sin 2x$
$+\int$	2	$-\frac{1}{4}\cos 2x$

$$\begin{aligned}\int x^2\cos(2x)\,dx &= \frac{1}{2}x^2\sin 2x+\frac{1}{2}x\cos 2x-\frac{1}{2}\int\cos 2x\ dx\\ &= \frac{1}{2}x^2\sin 2x+\frac{1}{2}x\cos 2x-\frac{1}{4}\sin 2x+C\end{aligned}$$

4. Use an appropriate trigonometric substitution to rewrite the following integral as a trigonometric integral in terms of the variable θ. Do not evaluate the integral, but do simplify the integrand as much as possible.

$$\int\frac{5}{x\sqrt{9-4x^2}}\ dx$$

$$\begin{aligned}2x &= 3\sin\theta\\ x &= \frac{3}{2}\sin\theta\\ \frac{dx}{d\theta} &= \frac{3}{2}\cos\theta\\ dx &= \frac{3}{2}\cos\theta\ d\theta\end{aligned}$$

$$\begin{aligned}\int\frac{5}{x\sqrt{9-4x^2}}\ dx &= \int\frac{5}{\left(\frac{3}{2}\sin\theta\right)\sqrt{9-4\left(\frac{3}{2}\sin\theta\right)^2}}\left(\frac{3}{2}\cos\theta\right)\ d\theta\\ &= \int\frac{5\cos\theta}{(\sin\theta)(3\cos\theta)}\ d\theta\\ &= \frac{5}{3}\int\csc\theta\ d\theta\end{aligned}$$

5. Evaluate $\int(\sin x)^2\tan x\ dx$

$$\begin{aligned}\int(\sin x)^2\tan x\ dx &= \int(\sin x)^2\frac{\sin x}{\cos x}\ dx\\ &= \int\sin x\frac{(\sin x)^2}{\cos x}\ dx\\ &= \int\sin x\frac{1-(\cos x)^2}{\cos x}\ dx\end{aligned}$$

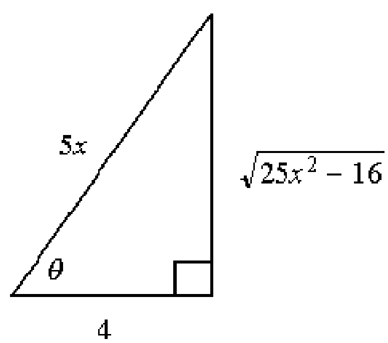
$$\begin{aligned}u &= \cos x\\ \frac{du}{dx} &= -\sin x\\ dx &= \frac{du}{-\sin x}\end{aligned}$$

$$
\begin{aligned}
\int (\sin x)^2 \tan x \; dx &= -\int \frac{1-u^2}{u} \; du \\
&= \int \frac{u^2-1}{u} du \\
&= \int \left(u - \frac{1}{u}\right) \; du \\
&= \frac{1}{2}u^2 - \ln|u| + C \\
&= \frac{1}{2}(\cos x)^2 - \ln|\cos x| + C
\end{aligned}
$$

6. Short Answer.

(a) Johann, a student in a Calculus II class, is in the middle of completing a trigonometric substitution problem using the substitution $5x = 4\sec\theta$. After evaluating the integral, Johann arrived at the answer $\cos\theta + 2\tan\theta + \frac{1}{2}\theta + C$. Finish this problem. (That is, "go back to x's" and simplify the answer as much as possible.)

$$\sec\theta = \frac{5x}{4}$$

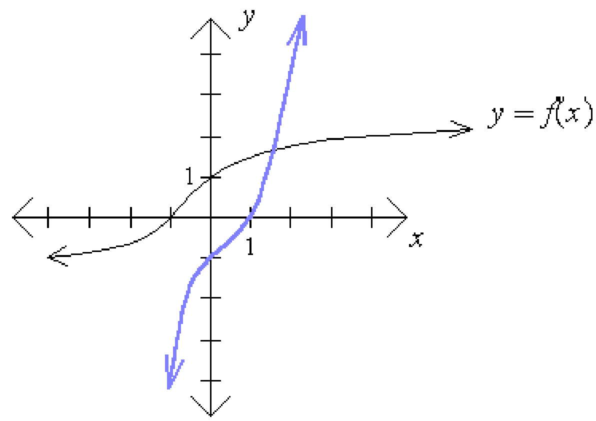


$$\cos\theta + 2\tan\theta + \frac{1}{2}\theta + C = \frac{4}{5x} + \frac{\sqrt{25x^2-16}}{2} + \frac{1}{2}\sec^{-1}\left(\frac{5x}{4}\right) + C$$

(b) Suppose $f(x)$ is a one-to-one function such that $f(2) = 7$, $f(7) = 1$, $f'(2) = 6$, and $f'(7) = 3$ and let $g = f^{-1}$. Find $g'(7)$.

$$g'(7) = \frac{1}{f'(g(7))} = \frac{1}{f'(2)} = \frac{1}{6}$$

(a) The graph of $f(x)$ is below. Sketch $y = f^{-1}(x)$ on the same axes.



(b) Find $\lim_{x\to\infty}\left(1-\frac{1}{x}\right)^x$

$$\begin{aligned}
\lim_{x\to\infty}\left(1-\frac{1}{x}\right)^x &= \lim_{x\to\infty} e^{\ln(1-1/x)^x} \\
&= \lim_{x\to\infty} e^{x\ln(1-1/x)} \\
&= e^{\lim_{x\to\infty} x\ln(1-1/x)} \\
&= e^{\lim_{x\to\infty}\frac{\ln(1-1/x)}{1/x}} \\
&\overset{\checkmark}{=} e^{\lim_{x\to\infty}\frac{\frac{1}{1-1/x}\left(\frac{1}{x^2}\right)}{-1/x^2}} \\
&= e^{\lim_{x\to\infty}\frac{-1}{1-1/x}} \\
&= e^{-1} \\
&= \frac{1}{e}
\end{aligned}$$

(a) Evaluate $\int\limits_{x=0}^{x=\pi/3} (\sin 3x)^2 \; dx$

$$\begin{aligned}
\int\limits_{x=0}^{x=\pi/3} (\sin 3x)^2 \; dx &= \int\limits_{x=0}^{x=\pi/3} \left(\frac{1}{2} - \frac{1}{2}\cos 6x\right) \; dx \\
&= \left.\frac{1}{2}x - \frac{1}{12}\sin 6x\right|_{x=0}^{x=\pi/3} \\
&= \left(\frac{\pi}{6} - 0\right) - (0) \\
&= \frac{\pi}{6}
\end{aligned}$$

(b) Give an example of a function $f(x)$ that has no inverse function.

$$f(x) = x^2$$

Many other answers possible.