1. Evaluate the integral $\int \frac{x^3}{x^2+4x}dx$

$$\frac{x^3}{x^2+4x} = \frac{x^2}{x+4}$$

$$\begin{array}{r l}
 & \underline{\;x-4\qquad\;} \\
x+4 &)x^2 \\
 & \underline{x^2+4x} \\
 & \quad -4x \\
 & \quad \underline{-4x-16} \\
 & \qquad\qquad 16
\end{array}$$

$$\begin{aligned}
\int \frac{x^3}{x^2+4x}dx &= \int \left(x-4+\frac{16}{x+4}\right)dx \\
&= \frac{1}{2}x^2-4x+16\ln|x+4|+C
\end{aligned}$$

2. Evaluate the integral $\int \frac{x}{(x+2)(x-3)}dx$

$$\begin{aligned}
\frac{x}{(x+2)(x-3)} &= \frac{A}{x+2}+\frac{B}{x-3} \\
x &= A(x-3)+B(x+2)
\end{aligned}$$

$$\begin{aligned}
x=3: & \\
3 &= 5B \Longrightarrow B=\frac{3}{5} \\
x=-2: & \\
-2 &= -5A \Longrightarrow A=\frac{2}{5}
\end{aligned}$$

$$\begin{aligned}
\int \frac{x}{(x+2)(x-3)}dx &= \int \left(\frac{2}{5}\left(\frac{1}{x+2}\right)+\frac{3}{5}\left(\frac{1}{x-3}\right)\right)dx \\
&= \frac{2}{5}\ln|x+2|+\frac{3}{5}\ln|x-3|+C
\end{aligned}$$

3. Either evaluate the integral $\int_{x=1}^{x=\infty} \frac{1}{(x-1)^3} dx$ or show that it diverges.

$$\begin{aligned}
\int_{x=1}^{x=\infty} \frac{1}{(x-1)^3} dx &= \int_{x=1}^{x=2} \frac{1}{(x-1)^3} dx + \int_{x=2}^{x=\infty} \frac{1}{(x-1)^3} dx \\
&= \lim_{t \to 1^+} \int_{x=t}^{x=2} \frac{1}{(x-1)^3} dx + \lim_{t \to \infty} \int_{x=2}^{x=t} \frac{1}{(x-1)^3} dx \\
&= \lim_{t \to 1^+} \left(\left. \frac{-1}{2(x-1)^2} \right|_{x=t}^{x=2} \right) + \lim_{t \to \infty} \left(\left. \frac{-1}{2(x-1)^2} \right|_{x=2}^{x=t} \right) \\
&= \lim_{t \to 1^+} \left(\frac{-1}{2} + \frac{1}{2(t-1)^2} \right) + \lim_{t \to \infty} \left(\frac{-1}{2(t-1)^2} + \frac{1}{2} \right)
\end{aligned}$$

Since this limit does not exist, the integral diverges.

4. Let a curve be described by the parametric equations $x = 3t + 5e^t, y = t + \sqrt{t+1}$. Find the tangent line to this curve at the point where $t = 0$.

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{1 + \frac{1}{2\sqrt{t+1}}}{3 + 5e^t}$$

$$m = \left. \frac{1 + \frac{1}{2\sqrt{t+1}}}{3 + 5e^t} \right|_{t=0} = \frac{1 + 1/2}{3 + 5} = \frac{3/2}{8} = \frac{3}{16}$$

At $t = 0$: $\quad x = 5, y = 1$

$$\begin{aligned}
\text{The tangent line is } y - 1 &= \frac{3}{16}(x - 5) \\
y &= \frac{3}{16}x + \frac{1}{16}
\end{aligned}$$

5. Find $\frac{d^2y}{dx^2}$ for the curve described by the parametric equations $x = t^3 - 4, y = 3t^2 + t + 1$ and then determine if the curve is concave up or concave down at the point where $t = 1$.

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{6t + 1}{3t^2}$$

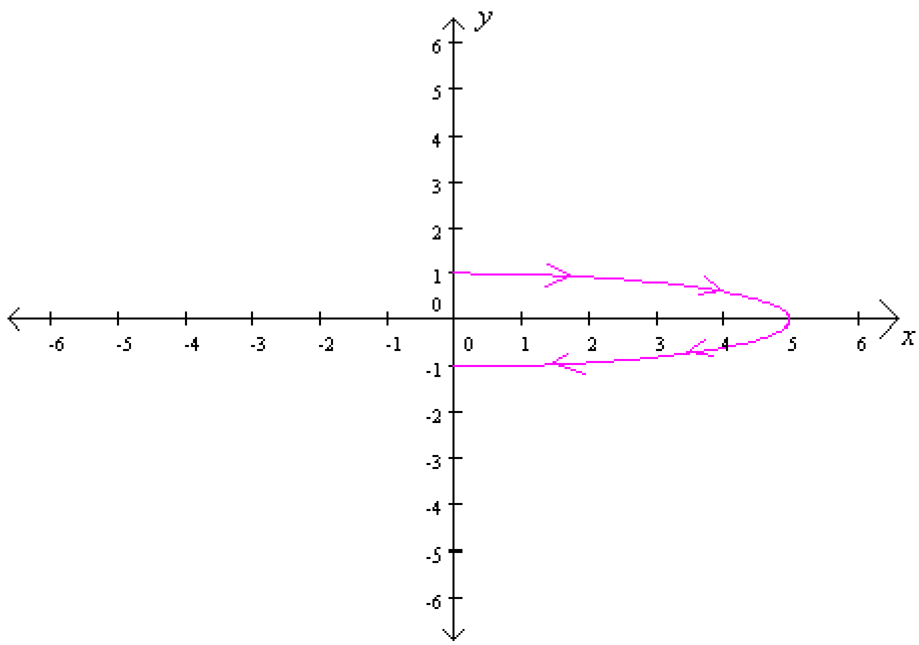
$$\frac{d^2y}{dx^2} = \frac{\left(\frac{d(dy/dx)}{dt} \right)}{dx/dt} = \frac{\left(\frac{3t^2(6) - (6t+1)6t}{9t^4} \right)}{3t^2} = \frac{-18t^2 - 6t}{27t^6} = \frac{-6t - 2}{9t^5}$$

$$\left.\frac{d^2y}{dx^2}\right|_{t=1} = \left.\frac{-6t-2}{9t^5}\right|_{t=1} = -\frac{8}{9}$$

The curve is concave down at this point.

6. Consider the curve described by the parametric equations $x = 5\sin t, y = \cos t, 0 \le t \le \pi$.

   (a) Sketch this curve on $xy-$axes. Demonstrate with arrows the direction of increasing t.

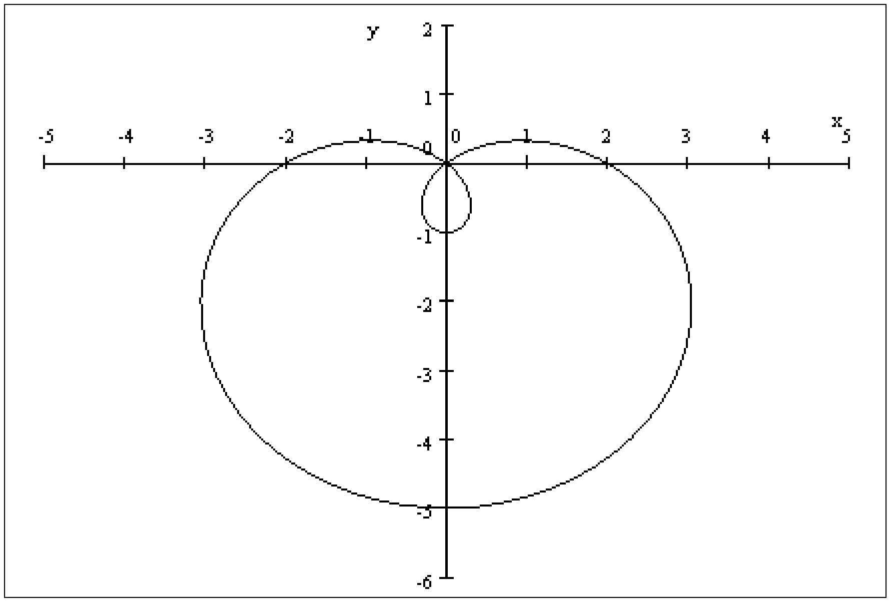


   (b) Eliminate the parameter to express the curve in terms of x and y. Simplify your answer as much as possible.

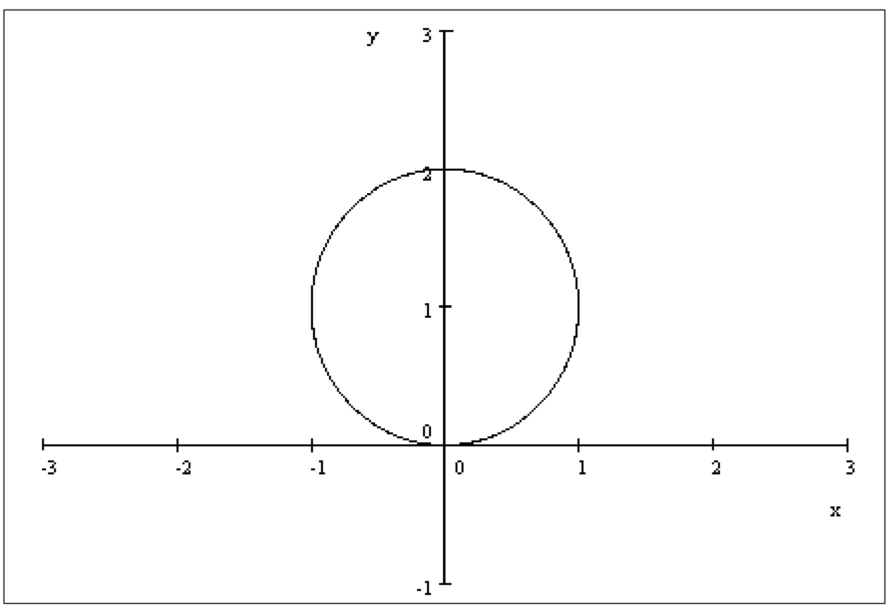
$$\sin t = \frac{x}{5}, \qquad \cos t = y$$

$$\begin{aligned}
(\sin t)^2 + (\cos t)^2 &= 1 \\
\left(\frac{x}{5}\right)^2 + y^2 &= 1 \\
\frac{x^2}{25} + y^2 &= 1
\end{aligned}$$

7. Sketch the polar curve $r = 2 - 3\sin\theta$ on $xy-$axes.



8. Find all points where the polar curve $r = 2\sin\theta$ has either a horizontal or vertical tanget line when sketched on xy-axes.



$$x = r\cos\theta = 2\sin\theta\cos\theta$$
$$y = r\sin\theta = 2(\sin\theta)^2$$
$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$$
$$= \frac{4\sin\theta\cos\theta}{2(\cos\theta)^2 - 2(\sin\theta)^2}$$

Horizontal:

$$\frac{dy}{dx} = 0$$

$$\frac{4\sin\theta\cos\theta}{2(\cos\theta)^2 - 2(\sin\theta)^2} = 0$$

$$4\sin\theta\cos\theta = 0$$

$$\theta = 0, \frac{\pi}{2}$$

Vertical:

$$\frac{dy}{dx} \text{ undefined}$$

$$2(\cos\theta)^2 - 2(\sin\theta)^2 = 0$$

$$(\cos\theta)^2 - (\sin\theta)^2 = 0$$

$$(\cos\theta)^2 = (\sin\theta)^2$$

$$\cos\theta = \pm\sin\theta$$

$$\theta = \frac{\pi}{4}, \frac{3\pi}{4}$$

The points where the curve is horizontal are $(0,0)$ and $(0,2)$.
The points where it is vertical are $(1,1)$ and $(-1,1)$