Math 152 - Lab 4 on the Calculus of Taylor Series. Due Wednesday.

Approximations with Taylor series

1. (a) Use the first 3 terms of the Maclaurin series expansion for e^x to estimate $e^{0.1}$ by hand.

   (b) Compute the value of $e^{0.1}$ using EasyCalc and compare to your approximation in part (a). How far off are the two results?

2. (a) Use the first 3 terms of the Maclaurin series expansion for $\cos x$ to estimate $\cos 0.1$ by hand.

   (b) Compute the value of $\cos 0.1$ using EasyCalc (make sure you're in radian mode) and compare to your approximation in part (a). How far off are the two results?

Derivatives with Taylor Series

1. (a) Write out the Maclaurin series for $\sin x$ using both the term-by-term form and the $\sum$ form.

   (b) Compute the derivative (with respect to x) of each of the two forms above and simplify as much as possible.

   (c) How does your result in part (b) compare to the Maclaurin series for $\cos x$? Explain why this makes sense.

2. (a) Find the derivative of the Maclaurin series (both forms) for e^x.

   (b) How does your result in part (a) compare to the Maclaurin series for e^x?

3. (a) Write the Maclaurin series (both forms) for $\frac{1}{1-x}$.

   (b) Differentiate both forms of the series.

   (c) What is the function that your series in part (b) represents?

Integrals with Taylor Series

1. (a) Explain why we cannot compute $\int_{x=0}^{x=1} e^{\left(x^2\right)} dx$ using any methods you learned from Calculus I.

   (b) Find the Maclaurin series representation for $e^{\left(x^2\right)}$. (Start with the Maclaurin series for e^x and use substitution.)

   (c) Now, find a Maclaurin series representation for $\int e^{\left(x^2\right)} dx$.

   (d) Use part (c) to find $\int_{x=0}^{x=1} e^{\left(x^2\right)} dx$. (Use enough terms that you are certain the first three decimal places are correct.)

2. Use series to find each of the following correct to the third decimal place:

   (a) $\int_{x=0}^{x=1} \sin(x^2) dx$

   (b) $\int_{x=-0.1}^{x=0.1} \frac{1}{1-x} dx$