Math 152 Name: KEY
Quiz 2

1. Let $a_n = \dfrac{5}{3^n}$, $n \geq 1$.

   (a) Does the sequence $\{a_n\}$ converge or diverge? If it converges, what does it converge to?

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{\overbrace{5}^{\substack{5\\ \uparrow}}}{\underbrace{3^n}_{\substack{\downarrow\\ \infty}}} = 0$$

The sequence converges to 0.

   (b) Does the series $\sum_{n=1}^{n=\infty} a_n$ converge or diverge? If it converges, what does it converge to?

$$\sum_{n=1}^{n=\infty} a_n = \sum_{n=1}^{n=\infty} \frac{5}{3^n} = \overbrace{\sum_{n=1}^{n=\infty} 5\left(\frac{1}{3}\right)^n}^{\substack{r=\frac{1}{3}\\ -1<\frac{1}{3}<1}} = \frac{5\left(\frac{1}{3}\right)}{1-\frac{1}{3}} = \frac{5/3}{2/3} = \frac{5}{2}$$

The series converges to $\dfrac{5}{2}$.

2. Determine if the series $\sum_{n=1}^{n=\infty} \dfrac{e^n}{2e^n + n}$ converges or diverges. You must fully justify your answer for credit.

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{\overbrace{e^n}^{\substack{\infty\\ \uparrow}}}{\underbrace{2e^n + n}_{\substack{\downarrow\\ \infty}}} \stackrel{\checkmark}{=} \lim_{n\to\infty} \frac{\overbrace{e^n}^{\substack{\infty\\ \uparrow}}}{\underbrace{e^n}_{\substack{\downarrow\\ \infty}}} = 1$$

By the Test for Divergence, $\sum_{n=1}^{n=\infty} \dfrac{e^n}{2e^n + n}$ diverges.

3. Use the fact that $\sum_{n=1}^{n=\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ to find $\sum_{n=3}^{n=\infty} \frac{1}{n^2}$.

$$\begin{aligned}
\sum_{n=3}^{n=\infty} \frac{1}{n^2} &= \left(\sum_{n=1}^{n=\infty} \frac{1}{n^2}\right) - \frac{1}{1^2} - \frac{1}{2^2} \\
&= \frac{\pi^2}{6} - 1 - \frac{1}{4} \\
&= \frac{\pi^2}{6} - \frac{5}{4}
\end{aligned}$$