Math 152 Name: KEY

Quiz 3

Your answers must be fully justified and unambiguous.

1. Does the series $\sum_{n=1}^{n=\infty} \frac{(-1)^n 2n}{n^2+1}$ converge absolutely, converge conditionally, or diverge?

First let's try the A.S.T. :

$$(i) \qquad \lim_{n\to\infty} \left| \frac{(-1)^n 2n}{n^2+1} \right| = \lim_{n\to\infty} \frac{2n}{n^2+1} \overset{\checkmark}{=} \lim_{n\to\infty} \frac{2}{2n} = 0 \qquad \checkmark$$

$$(ii) \qquad |a_{n+1}| \overset{?}{\leq} |a_n|$$

$$\frac{2(n+1)}{(n+1)^2+1} \overset{?}{\leq} \frac{2n}{n^2+1}$$

$$\frac{2n+2}{n^2+2n+2} \overset{?}{\leq} \frac{2n}{n^2+1}$$

$$2n^3+2n^2+2n+2 \overset{?}{\leq} 2n^3+4n^2+4n$$

$$0 \overset{?}{\leq} 2n^2+2n-2 \qquad \text{(True for } n \geq 1) \qquad \checkmark$$

So, $\sum_{n=1}^{n=\infty} \frac{(-1)^n 2n}{n^2+1}$ converges, but is it absolute convergence or conditional convergence.

We consider the absolute value series.

The absolute value series is $\sum_{n=1}^{n=\infty} \frac{2n}{n^2+1}$. We compare this series to $\sum_{n=1}^{n=\infty} \frac{1}{n}$, which diverges (Harmonic Series).

$$\lim_{n\to\infty} \frac{\left(\frac{2n}{n^2+1}\right)}{\left(\frac{1}{n}\right)} = \lim_{n\to\infty} \frac{2n^2}{n^2+1} \overset{\checkmark}{=} \lim_{n\to\infty} \frac{4n}{2n} = 2.$$

Thus, $\sum_{n=1}^{n=\infty} \frac{2n}{n^2+1}$ diverges by the O.C.T.

Therefore, $\sum_{n=1}^{n=\infty} \frac{(-1)^n 2n}{n^2+1}$ converges conditionally.

2. Does the series $\sum_{n=1}^{n=\infty} \frac{(-1)^n 5^n}{5^n+3}$ converge absolutely, converge conditionally, or diverge?

$$\lim_{n\to\infty} \left| \frac{(-1)^n 5^n}{5^n+3} \right| = \lim_{n\to\infty} \frac{5^n}{5^n+3} \overset{\checkmark}{=} \lim_{n\to\infty} \frac{5^n \ln 5}{5^n \ln 5} = 1$$

Thus, $\sum_{n=1}^{n=\infty} \frac{(-1)^n 5^n}{5^n+3}$ diverges by the T.F.D.