Math 152 Quiz 4 Name: KEY

Your answers must be fully justified.

1. Find the Taylor series for $f(x) = \dfrac{1}{x}$, centered at $x = 1$. (For full credit, you must write both the term-by-term form and the sigma notation form of the series and simplify your answers as much as possible. Show at least 5 non-zero terms in your term-by-term series.)

$f(x) = \dfrac{1}{x}$	$f(1) = 1$	$c_0 = \dfrac{1}{0!} = 1$
$f'(x) = -\dfrac{1}{x^2}$	$f'(1) = -1$	$c_1 = \dfrac{-1}{1!} = -1$
$f''(x) = \dfrac{2}{x^3}$	$f''(1) = 2$	$c_2 = 1$
$f'''(x) = -\dfrac{6}{x^4}$	$f'''(1) = -6$	$c_3 = -1$
$f^{(4)}(x) = \dfrac{24}{x^7}$	$f^{(4)}(1) = 24$	$c_4 = 1$
$\vdots$	$\vdots$	$\vdots$

$$f(x) = 1 - (x-1) + (x-1)^2 - (x-1)^3 + (x-1)^4 - \ldots = \sum_{n=0}^{n=\infty} (-1)^n (x-1)^n$$

2. Find (or state) the Maclaurin series for each of the following. (For full credit, you must write both the term-by-term form and the sigma notation form of the series and simplify your answers as much as possible. Show at least 5 non-zero terms in your term-by-term series.)

(a) $\cos x$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \ldots = \sum_{n=0}^{n=\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

(b) $\cos(-5x)$

$$\begin{aligned} \cos(-5x) &= 1 - \frac{(-5x)^2}{2!} + \frac{(-5x)^4}{4!} - \frac{(-5x)^6}{6!} + \frac{(-5x)^8}{8!} - \ldots = \sum_{n=0}^{n=\infty} \frac{(-1)^n (-5x)^{2n}}{(2n)!} \\ &= 1 - \frac{5^2x^2}{2!} + \frac{5^4x^4}{4!} - \frac{5^6x^6}{6!} + \frac{5^8x^8}{8!} - \ldots = \sum_{n=0}^{n=\infty} \frac{(-1)^n 5^{2n} x^{2n}}{(2n)!} \end{aligned}$$

(c) $x^3 \cos(-5x)$

$$\begin{aligned} x^3 \cos(-5x) &= x^3 \left(1 - \frac{5^2x^2}{2!} + \frac{5^4x^4}{4!} - \frac{5^6x^6}{6!} + \frac{5^8x^8}{8!} - \ldots\right) = x^3 \cdot \sum_{n=0}^{n=\infty} \frac{(-1)^n 5^{2n} x^{2n}}{(2n)!} \\ &= x^3 - \frac{5^2x^5}{2!} + \frac{5^4x^7}{4!} - \frac{5^6x^9}{6!} + \frac{5^8x^{11}}{8!} - \ldots = \sum_{n=0}^{n=\infty} \frac{(-1)^n 5^{2n} x^{2n+3}}{(2n)!} \end{aligned}$$