

Your answers must be fully justified to receive credit.

1. Let $f(x) = \arcsin\left(\frac{x^2}{4}\right)$. Find $f'(1)$.

$$\begin{aligned} f'(x) &= \frac{1}{\sqrt{1 - \left(\frac{x^2}{4}\right)^2}} \left(\frac{2x}{4}\right) \\ &= \frac{x}{2\sqrt{1 - \frac{x^4}{16}}} \\ f'(1) &= \frac{1}{2\sqrt{1 - \frac{1}{16}}} = \frac{1}{2\sqrt{\frac{15}{16}}} = \frac{2}{\sqrt{15}} \end{aligned}$$

2. Simplify each of the following as much as possible:

(a) $\log_3 \frac{3}{2} + \log_3 30 - \log_3 5$

$$\begin{aligned} \log_3 \frac{3}{2} + \log_3 30 - \log_3 5 &= \log_3 \left(\frac{\left(\frac{3}{2}\right)(30)}{5} \right) \\ &= \log_3 9 \\ &= 2 \end{aligned}$$

(b) $\ln e^4 + e^{\ln 7}$

$$\ln e^4 + e^{\ln 7} = 4 + 7 = 11$$

3. Find $f'(x)$.

(a) $f(x) = x^3 \ln(4x + 1)$

$$\begin{aligned} f'(x) &= 3x^2 \ln(4x + 1) + x^3 \frac{1}{4x + 1} (4) \\ &= 3x^2 \ln(4x + 1) + \frac{4x^3}{4x + 1} \end{aligned}$$

(b) $f(x) = \ln \frac{(\sqrt{x^2 + 3})(x + 1)}{x^3 - 4x + 2}$. (Hint: Rewrite $f(x)$ first.)

$$\begin{aligned} f(x) &= \ln(\sqrt{x^2 + 3}) + \ln(x + 1) - \ln(x^3 - 4x + 2) \\ &= \frac{1}{2} \ln(x^2 + 3) + \ln(x + 1) - \ln(x^3 - 4x + 2) \\ f'(x) &= \frac{1}{2} \frac{1}{x^2 + 3} (2x) + \frac{1}{x + 1} - \frac{1}{x^3 - 4x + 2} (3x^2 - 4) \\ &= \frac{x}{x^2 + 3} + \frac{1}{x + 1} - \frac{3x^2 - 4}{x^3 - 4x + 2} \end{aligned}$$