

Evaluate the integral.

$$1. \int_{x=0}^{x=\pi/2} (\sin x)^4 (\cos x)^3 dx$$

$$\begin{aligned} \int (\sin x)^4 (\cos x)^3 dx &= \int \cos x (\sin x)^4 (\cos x)^2 dx \\ &= \int \cos x (\sin x)^4 (1 - (\sin x)^2) dx \end{aligned}$$

$$\begin{aligned} u &= \sin x \\ \frac{du}{dx} &= \cos x \\ dx &= \frac{du}{\cos x} \end{aligned}$$

$$\begin{aligned} \int (\sin x)^4 (\cos x)^3 dx &= \int u^4 (1 - u^2) du \\ &= \int (u^4 - u^6) du \\ &= \frac{1}{5}u^5 - \frac{1}{7}u^7 + C \\ &= \frac{1}{5}(\sin x)^5 - \frac{1}{7}(\sin x)^7 + C \end{aligned}$$

$$\begin{aligned} \int_{x=0}^{x=\pi/2} (\sin x)^4 (\cos x)^3 dx &= \left. \frac{1}{5}(\sin x)^5 - \frac{1}{7}(\sin x)^7 \right]_{x=0}^{x=\pi/2} \\ &= \left(\frac{1}{5} - \frac{1}{7} \right) - 0 \\ &= \frac{2}{35} \end{aligned}$$

$$2. \int (\cos(5x))^2 dx$$

$$\begin{aligned} \int (\cos(5x))^2 dx &= \int \left(\frac{1}{2} + \frac{1}{2} \cos(10x) \right) dx \\ &= \frac{1}{2}x + \frac{1}{20} \sin(10x) + C \end{aligned}$$