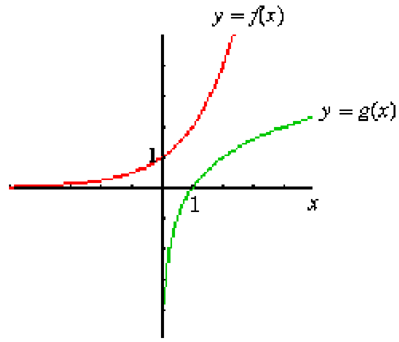
Math 152 - Worksheet on Inverse Exponential Functions

1. Let $f(x) = 2^x$ and $g = f^{-1}$.

   (a) Sketch f and g on the same axes.



   (b) Evaluate $g(1), g(2), g(4), g(8)$, and $g(64)$.

$$g(1) = 0, g(2) = 1, g(4) = 2, g(8) = 3, g(64) = 4$$

   (c) Evaluate $g\left(\frac{1}{2}\right), g\left(\frac{1}{4}\right), g\left(\frac{1}{8}\right), g\left(\frac{1}{64}\right)$.

$$g\left(\frac{1}{2}\right) = -1, g\left(\frac{1}{4}\right) = -2, g\left(\frac{1}{8}\right) = -3, g\left(\frac{1}{64}\right) = -4$$

   (d) Evaluate $g\left(\sqrt{2}\right), g\left(\sqrt[5]{2}\right), g\left(\frac{1}{\sqrt{2}}\right), g\left(\frac{1}{\sqrt[5]{2}}\right)$

$$g\left(\sqrt{2}\right) = \frac{1}{2}, g\left(\sqrt[5]{2}\right) = \frac{1}{5}, g\left(\frac{1}{\sqrt{2}}\right) = -\frac{1}{2}, g\left(\frac{1}{\sqrt[5]{2}}\right) = -\frac{1}{5}$$

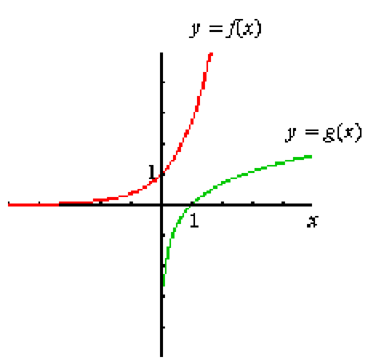
   (e) Evaluate $2^{g(8)}, 2^{g(1/2)}, 2^{g(1)}$.

$$2^{g(8)} = 8, 2^{g(1/2)} = \frac{1}{2}, 2^{g(1)} = 1$$

   (f) For any number x, $g(2^x) =$ ___x___.

2. Let $f(x) = e^x$ and $g = f^{-1}$.

   (a) Sketch f and g on the same axes.



(b) Evaluate $g(1), g(e), g(e^2), g(e^3)$, and $g\left(e^{27}\right)$.

$$g(1) = 0, g(e) = 1, g(e^2) = 2, g(e^3) = 3, g\left(e^{27}\right) = 27$$

(c) Evaluate $g\left(\frac{1}{e}\right), g\left(\frac{1}{e^2}\right), g\left(\sqrt{e}\right), g\left(\sqrt[7]{e^4}\right)$.

$$g\left(\frac{1}{e}\right) = -1, g\left(\frac{1}{e^2}\right) = -2, g\left(\sqrt{e}\right) = \frac{1}{2}, g\left(\sqrt[7]{e^4}\right) = \frac{4}{7}$$

(d) Evaluate $e^{g(e)}, e^{g(1/e)}, e^{g(1)}$.

$$e^{g(e)} = e, e^{g(1/e)} = \frac{1}{e}, e^{g(1)} = 1$$

(e) For any number x, $g(e^x) =$ ___x___.

3. If $f(x) = a^x$ and $g = f^{-1}$, then, for any number x, $g\left(a^x\right) =$ ___x___ and $a^{g(x)} =$ ___x___

4. Let $f(x) = 3^x$ and $g = f^{-1}$.

(a) Find $g(9) + g(3)$ and $g(27)$. (Note: $9 * 3 = 27$)

$$g(9) + g(3) = 2 + 1 = 3, g(27) = 3$$

(b) Find $g(1) + g(81)$ and $g(81)$.

$$g(1) + g(81) = 0 + 4 = 4, g(81) = 4$$

(c) Find $g\left(\frac{1}{3}\right) + g(9)$ and $g(3)$.

$$g\left(\frac{1}{3}\right) + g(9) = -1 + 2 = 1, g(3) = 3$$

(d) Based on these observations, $g(a) + g(b) =$ ___$g(ab)$___.

5. Let $f(x) = 5^x$ and $g = f^{-1}$.

(a) Find $g\left(25^3\right)$ and find $3g(25)$.

$$g\left(25^3\right) = g\left(\left(5^2\right)^3\right) = g\left(5^6\right) = 6, 3g(25) = 3 \cdot 2 = 6$$

(b) Find $g\left(5^7\right)$ and find $7g(5)$.

$$g\left(5^7\right) = 7, 7g(5) = 7 \cdot 1 = 7$$

(c) Find $g\left(\sqrt{5}\right)$ and find $\frac{1}{2}g(5)$.

$$g\left(\sqrt{5}\right) = \frac{1}{2}, \frac{1}{2}g(5) = \frac{1}{2} \cdot 1 = \frac{1}{2}$$

(d) Based on these observations, $g(n^m) =$ $\underline{\quad m \cdot g(n) \quad}$.

6. If $f(x) = a^x$, then we write $\ln_a(x)$ for the inverse function. That is, in place of g in all of the problems above, we could have written $\log_a$, where a is the base of the original exponential functon f.

   (a) $\log_a(a) =$ $\underline{\quad 1 \quad}$.

   (b) $\log_a(a^m) =$ $\underline{\quad m \quad}$.

   (c) $a^{\log_a(a^m)} =$ $\underline{\quad a^m \quad}$.

   (d) $\log_a(b) + \log_a(c) =$ $\underline{\quad \log_a(b \cdot c) \quad}$.

   (e) $m \log_a(x) =$ $\underline{\quad \log_a(x^m) \quad}$

   (f) $-\log_a(x) =$ $\underline{\quad \log_a\left(\frac{1}{x}\right) \quad}$

   (g) $\log_a(b) - \log_a(c) =$ $\underline{\quad \log_a\left(\frac{b}{c}\right) \quad}$.

   (h) If $y = \log_a(x)$, then $x =$ $\underline{\quad a^y \quad}$.

   (i) If $a > 1$, then $\lim_{x \to \infty} \log_a(x) =$ $\underline{\quad \infty \quad}$.

   (j) If $a > 1$, then $\lim_{x \to 0^+} \log_a(x) =$ $\underline{\quad -\infty \quad}$.

   (k) $\log_a(1) =$ $\underline{\quad 0 \quad}$.

   (l) $\log_a(x) =$ "the number that you have to raise a to in order to get x" or "the exponent that you put on a to get x" or "the inverse function of $y = a^x$".

Note: When $a = 10$, we write log instead. So $\log(x) = \log_{10}(x)$. When $a = e$, we write ln instead. So $\ln(x) = \log_e(x)$.