Math 152 Quiz 6 Name: KEY

Your answers must be fully justified to receive credit.

1. Simplify each of the following as much as possible:

   (a) $\log_3 \frac{3}{2} + \log_3 24 - \log_3 4$

$$\log_3 \frac{3}{2} + \log_3 24 - \log_3 4 = \log_3 \left(\frac{3}{2} \cdot 24 \div 4\right) = \log_3 9 = \log_3 3^2 = 9$$

   (b) $\ln e^5 + e^{\ln 6} + e^{3\ln 2}$

$$\ln e^5 + e^{\ln 6} + e^{3\ln 2} = 5 + 6 + e^{\ln 2^3} = 11 + 2^3 = 19$$

2. Find $f'(x)$.

   (a) $f(x) = \ln \frac{\left(\sqrt{x^2+3}\right)(x+1)}{x^3-4x+2}$ (Hint: Rewrite it first.)

$$\begin{aligned} f(x) &= \ln \frac{\left(\sqrt{x^2+3}\right)(x+1)}{x^3-4x+2} \\ &= \ln \sqrt{x^2+3} + \ln(x+1) - \ln\left(x^3-4x+2\right) \\ &= \frac{1}{2}\ln\left(x^2+3\right) + \ln(x+1) - \ln\left(x^3-4x+2\right) \\ f'(x) &= \frac{1}{2}\frac{1}{x^2+3}\cdot 2x + \frac{1}{x+1} - \frac{1}{x^3-4x+2}\cdot\left(3x^2-4\right) \\ &= \frac{x}{x^2+3} + \frac{1}{x+1} - \frac{3x^2-4}{x^3-4x+2} \end{aligned}$$

   (b) $f(x) = x^{2x+1}$

$$\begin{aligned} f(x) &= x^{2x+1} \\ &= e^{\ln\left(x^{2x+1}\right)} \\ &= e^{(2x+1)\ln x} \\ f'(x) &= e^{(2x+1)\ln x}\left(2\ln x + (2x+1)\cdot\frac{1}{x}\right) \\ &= x^{2x+1}\left(2\ln x + 2 + \frac{1}{x}\right) \end{aligned}$$

3. Find $\lim_{x\to\infty} (e^{2x}+4)^{3/x}$

$$\begin{aligned}
\lim_{x\to\infty} \left(e^{2x}+4\right)^{3/x} &= \lim_{x\to\infty} e^{\ln\left(e^{2x}+4\right)^{3/x}} \\
&= \lim_{x\to\infty} e^{\frac{3}{x}\ln\left(e^{2x}+4\right)} \\
&= \lim_{x\to\infty} e^{\frac{3\ln\left(e^{2x}+4\right)}{x}} \\
&= e^{\lim_{x\to\infty} \frac{3\ln\left(e^{2x}+4\right)}{x}} \\
&\stackrel{\checkmark}{=} e^{\lim_{x\to\infty} \frac{\frac{3}{e^{2x}+4}\cdot 2e^{2x}}{1}} \\
&= e^{\lim_{x\to\infty} \frac{6e^{2x}}{e^{2x}+4}} \\
&\stackrel{\checkmark}{=} e^{\lim_{x\to\infty} \frac{12e^{2x}}{2e^{2x}}} \\
&= e^{6}
\end{aligned}$$