1. Find the equation of the osculating plane for the helix traced out by the vector function $\vec{r}(t) = \langle \sin t, \cos t, 2t \rangle$ at the point where $t = \dfrac{\pi}{2}$.

$$\begin{aligned}
\vec{r}\left(\frac{\pi}{2}\right) &= \langle 1, 0, \pi \rangle \quad \text{(Note: this gives us a point on the ocsulating plane.)} \\
\vec{r}'(t) &= \langle \cos t, -\sin t, 2 \rangle \\
\vec{r}'\left(\frac{\pi}{2}\right) &= \langle 0, -1, 2 \rangle \\
\vec{T}(t) &= \frac{\vec{r}'(t)}{|\vec{r}'(t)|} = \frac{\langle \cos t, -\sin t, 2 \rangle}{\sqrt{5}} \\
\vec{T}'(t) &= \frac{\langle -\sin t, -\cos t, 0 \rangle}{\sqrt{5}} \\
\vec{T}'\left(\frac{\pi}{2}\right) &= \frac{\langle -1, 0, 0 \rangle}{\sqrt{5}} \\
\vec{N}\left(\frac{\pi}{2}\right) &= \frac{\vec{T}'\left(\frac{\pi}{2}\right)}{\left|\vec{T}'\left(\frac{\pi}{2}\right)\right|} = \langle -1, 0, 0 \rangle
\end{aligned}$$

We want a vector that is normal to the osculating plane. We can use either $\vec{T}\left(\frac{\pi}{2}\right) \times \vec{N}\left(\frac{\pi}{2}\right)$ or $\vec{r}'\left(\frac{\pi}{2}\right) \times \vec{N}\left(\frac{\pi}{2}\right)$, etc. Let's use $\vec{r}'\left(\frac{\pi}{2}\right) \times \vec{N}\left(\frac{\pi}{2}\right)$ because these are the simplest vectors.

$$\vec{r}'\left(\frac{\pi}{2}\right) \times \vec{N}\left(\frac{\pi}{2}\right) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & -1 & 2 \\ -1 & 0 & 0 \end{vmatrix} = 2\vec{j} - \vec{k}$$

Thus, the osculating plane is $2y - (z - \pi) = 0$.

2. The acceleration of a particular particle moving in space is given by $\vec{a}(t) = 3\vec{i} + \vec{j} - 2t\vec{k}$ meters/second^2 after t seconds. When $t = 2$ seconds, the velocity vector of the particle is $\vec{i} + \vec{j} + \vec{k}$. Find the **speed** of the particle when $t = 3$ seconds.

$$\begin{aligned}
\vec{a}(t) &= 3\vec{i} + \vec{j} - 2t\vec{k} \\
\vec{v}(t) &= \int a(t)\,dt = 3t\vec{i} + t\vec{j} - t^2\vec{k} + \vec{C} \\
\text{So, } \vec{v}(2) &= 6\vec{i} + 2\vec{j} - 4\vec{k} + \vec{C}. \\
\text{On the other hand, } \vec{v}(2) &= \vec{i} + \vec{j} + \vec{k}. \\
\text{Thus, } \vec{C} &= -5\vec{i} - \vec{j} + 5\vec{k}. \\
\vec{v}(t) &= (3t - 5)\vec{i} + (t - 1)\vec{j} + \left(5 - t^2\right)\vec{k} \\
\vec{v}(3) &= 4\vec{i} + 2\vec{j} - 4\vec{k} \\
|\vec{v}(3)| &= 6 \text{ meters/second}
\end{aligned}$$