Math 251 Quiz 8 Name: KEY

1. Let $f(x,y) = x^2 \sin(xy) + 2y$.

   (a) Find the equation of the plane tangent to this surface at the point where $x = 1$ and $y = \frac{\pi}{2}$.

$$f\left(1, \frac{\pi}{2}\right) = 1 + \pi$$

$$\begin{aligned} f_x(x,y) &= 2x\sin(xy) + x^2 y\cos(xy) \\ f_x\left(1,\frac{\pi}{2}\right) &= 2 \\ f_y(x,y) &= x^3\cos(xy) + 2 \\ f_y\left(1,\frac{\pi}{2}\right) &= 2 \end{aligned}$$

$$\begin{aligned} \text{So, the tangent plane is} &: \\ 2(x-1) + 2\left(y - \frac{\pi}{2}\right) - (z - (1+\pi)) &= 0 \\ 2x + 2y - z &= 1 \end{aligned}$$

   (b) Find $f_{xy}(x,y)$.

$$\begin{aligned} f_{xy}(x,y) &= 2x^2\cos(xy) + x^2\cos(xy) - x^3 y\sin(xy) \\ &= 3x^3\cos(xy) - x^3 y \sin(xy) \end{aligned}$$

2. Let $f(x,y) = x\sqrt{y}$. Use differentials to estimate $f(3.01, 4.01)$. (Hint: $(3,4)$ is a "nice" point near $(3.01, 4.01)$.)

$$\begin{aligned} \triangle x &= 3.01 - 3 = 0.01 \\ \triangle y &= 4.01 - 4 = 0.01 \end{aligned}$$

$$\begin{aligned} f_x(x,y) &= \sqrt{y} \\ f_x(3,4) &= 2 \\ f_y(x,y) &= \frac{x}{2\sqrt{y}} \\ f_y(3,4) &= \frac{3}{4} = 0.75 \end{aligned}$$

$$\begin{aligned} dz &= f_x(3,4)\,dx + f_y(3,4)\,dy \\ &\approx f_x(3,4)\triangle x + f_y(3,4)\triangle y \\ &= (2)(0.01) + (0.75)(0.01) \\ &= 0.02 + 0.0075 \\ &= 0.0275 \end{aligned}$$

$$f(3.01, 4.01) \approx f(3,4) + dz = 6 + 0.0275 = 6.0275$$

(To 9 decimal places of accuracy, the value of $3.01\sqrt{4.01}$ is 6.027520303, so the above approximation is extremely good.)