Math 251 Quiz 9 Name: KEY

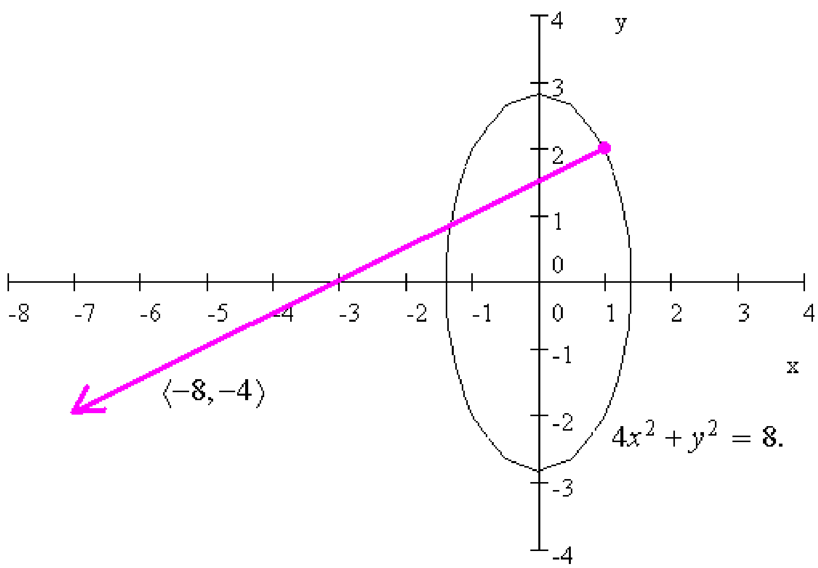
1. Let $f(x,y) = 24 - 4x^2 - y^2$.

(a) Find $\vec{\nabla} f(1,2)$.

$$\begin{aligned} \vec{\nabla} f(x,y) &= \langle f_x(x,y), f_y(x,y) \rangle \\ &= \langle -8x, -2y \rangle \\ \vec{\nabla} f(1,2) &= \langle -8, -4 \rangle \end{aligned}$$

(b) Make a careful sketch (on $xy-$axes) of the level curve that passes through the point $(1,2)$ along with $\vec{\nabla} f(1,2)$ situated with its tail at $(1,2)$.

$f(1,2) = 16$. So, we want the level curve for $k = 16$.
That is, we want the curve $24 - 4x^2 - y^2 = 16$,
which is the ellipse $4x^2 + y^2 = 8$.



2. (a) Verify that $(-1,0)$ and $(-2,-\frac{1}{2})$ are critical points of $f(x,y) = \frac{1}{4}x^4y^2 - 4xy - 4y$.

$$f_x(x,y) = x^3y^2 - 4y$$

$f_x(-1,0) = 0$ ✓ $\quad f_x\left(-2,-\frac{1}{2}\right) = -2 + 2 = 0$ ✓

$$f_y(x,y) = \frac{1}{2}x^4y - 4x - 4$$

$f_y(-1,0) = 4 - 4 = 0$ ✓ $\quad f_y\left(-2,-\frac{1}{2}\right) = -4 + 8 - 4 = 0$ ✓

Since both partial derivatives are 0 for both points, the points are critical points of the function.

(b) Classify each of these critical points as either a local maximum, local minimum, or saddle point.

$$\begin{aligned} f_{xx}(x,y) &= 3x^2y^2 \\ f_{xy}(x,y) &= 2x^3y - 4 \\ f_{yy}(x,y) &= \frac{1}{2}x^4 \end{aligned}$$

$$\begin{aligned} (-1,0) &: \\ D &= f_{xx}(-1,0)\, f_{yy}(-1,0) - (f_{xy}(-1,0))^2 \\ &= (0)\left(\frac{1}{2}\right) - (-4)^2 \\ &< 0 \\ &\text{So, } (-1,0) \text{ is a saddle point.} \end{aligned}$$

$$\begin{aligned} \left(-2,-\frac{1}{2}\right) &: \\ D &= f_{xx}\left(-2,-\frac{1}{2}\right) f_{yy}\left(-2,-\frac{1}{2}\right) - \left(f_{xy}\left(-2,-\frac{1}{2}\right)\right)^2 \\ &= (3)(8) - (4)^2 \\ &> 0 \\ &\text{and } f_{xx}\left(-2,-\frac{1}{2}\right) = 3 > 0 \\ &\text{So, } \left(-2,-\frac{1}{2}\right) \text{ is a local minimum.} \end{aligned}$$