1. Evaluate $\int_0^1 \frac{4x^5}{x^6+10} dx$

$$u = x^6 + 10$$
$$du = 6x^5 dx$$

$$\begin{aligned}\int \frac{4x^5}{x^6+10} dx &= \frac{2}{3} \int \frac{1}{u} du \\ &= \frac{2}{3} \ln |u| + C \\ &= \frac{2}{3} \ln \left| x^6 + 10 \right| + C\end{aligned}$$

$$\begin{aligned}\int_0^1 \frac{4x^5}{x^6+10} dx &= \left. \frac{2}{3} \ln \left| x^6 + 10 \right| \right]_0^1 \\ &= \frac{2}{3} \ln 11 - \frac{2}{3} \ln 10 \\ &= \frac{2}{3} \ln \left(\frac{11}{10} \right)\end{aligned}$$

2. Evaluate $\int x\, e^{3x}\, dx$

$$f(x) = x \qquad g'(x) = e^{3x}$$
$$f'(x) = 1 \qquad g(x) = \frac{1}{3} e^{3x}$$

$$\begin{aligned}\int x\, e^{3x}\, dx &= \frac{1}{3} x e^{3x} - \int \frac{1}{3} e^{3x} dx \\ &= \frac{1}{3} x e^{3x} - \frac{1}{9} e^{3x} + C\end{aligned}$$

3. Evaluate $\int \frac{x}{(x-1)^2(x+1)}dx$

$$\frac{x}{(x-1)^2(x+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1}$$

$$x = A(x-1)(x+1) + B(x+1) + C(x-1)^2$$

$$x = 1 : 1 = 2B \implies B = \frac{1}{2}$$

$$x = -1 : -1 = 4C \implies C = -\frac{1}{4}$$

$$x = 0 : 0 = -A + B + C \implies 0 = -A + \frac{1}{2} - \frac{1}{4} \implies A = \frac{1}{4}$$

$$\int \frac{x}{(x-1)^2(x+1)}dx = \int \left(\frac{\left(\frac{1}{4}\right)}{x-1} + \frac{\left(\frac{1}{2}\right)}{(x-1)^2} + \frac{\left(-\frac{1}{4}\right)}{x+1} \right) dx$$

$$= \frac{1}{4}\int \frac{1}{x-1}dx + \frac{1}{2}\int \frac{1}{(x-1)^2}dx - \frac{1}{4}\int \frac{1}{x+1}dx$$

$$= \frac{1}{4}\ln|x-1| - \frac{1}{2}\left(\frac{1}{x-1}\right) - \frac{1}{4}\ln|x+1| + C$$

4. Convert the integral $\int \frac{x^2}{\sqrt{25x^2-9}}dx$ into a trigonometric integral (i.e., an integral containing only trigonometric functions and only the variable θ).
Do not evaluate the trigonometric integral, but do simplify it as much as possible.

$$5x = 3\sec\theta$$

$$x = \frac{3}{5}\sec\theta$$

$$dx = \frac{3}{5}\sec\theta\tan\theta\, d\theta$$

$$\int \frac{x^2}{\sqrt{25x^2-9}}dx = \int \frac{\left(\frac{3}{5}\sec\theta\right)^2}{\sqrt{25\left(\frac{3}{5}\sec\theta\right)^2 - 9}} \frac{3}{5}\sec\theta\tan\theta\, d\theta$$

$$= \frac{9}{125}\int \frac{(\sec\theta)^3\tan\theta}{\tan\theta}d\theta$$

$$= \frac{9}{125}\int (\sec\theta)^3\, d\theta$$

5. Either evaluate $\int_{-2}^{2} \frac{1}{x^2}\,dx$, or show that it diverges.

$$\begin{aligned}
\int_{-2}^{2} \frac{1}{x^2}\,dx &= \int_{-2}^{0} \frac{1}{x^2}\,dx + \int_{0}^{2} \frac{1}{x^2}\,dx \\
&= \lim_{t\to 0^-} \int_{-2}^{t} \frac{1}{x^2}\,dx + \lim_{w\to 0^+} \int_{w}^{2} \frac{1}{x^2}\,dx \\
&= \lim_{t\to 0^-} \left(\left.\frac{-1}{x}\right]_{-2}^{t} \right) + \lim_{w\to 0^+} \left(\left.\frac{-1}{x}\right]_{w}^{2} \right) \\
&= \lim_{t\to 0^-} \left(\frac{-1}{t} - \frac{1}{2} \right) + \lim_{w\to 0^+} \left(\frac{-1}{2} + \frac{1}{w} \right)
\end{aligned}$$

Since at least one of the limits does not exist (actually, neither exists, but we only need one), the integral diverges.

6. Let $x = t^2 - 2t - 1, y = t^2 + 4t + 2$.

(a) Find all points on the curve where the tangent line is either horizontal or vertical.

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{2t+4}{2t-2} = \frac{t+2}{t-1}$$

horizontal:

$$t + 2 = 0$$
$$t = -2$$

$$(x, y) = (7, -2)$$

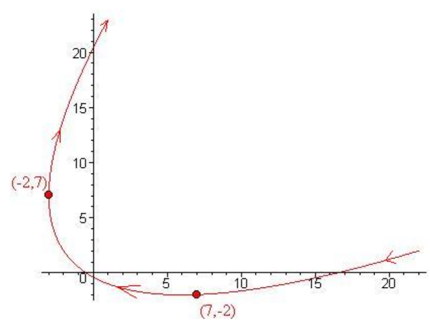
vertical:

$$t - 1 = 0$$
$$t = 1$$

$$(x, y) = -2, 7)$$

(b) Use an analysis of intervals to sketch the curve on $xy-$axes.

	$(-\infty, -2)$	$(-2, 1)$	$(1, \infty)$
$\frac{dy}{dt} = 2t + 4$	$-$	$+$	$+$
$\frac{dx}{dt} = 2t - 2$	$-$	$-$	$+$
curve	↙	↖	↗



7. (a) Convert the polar coordinates $(2, \frac{3\pi}{4})$ into rectangular coordinates.

$$\begin{aligned} x &= r\cos\theta = 2\cos\frac{3\pi}{4} = -\sqrt{2} \\ y &= r\sin\theta = 2\sin\frac{3\pi}{4} = \sqrt{2} \\ (x, y) &= (-\sqrt{2}, \sqrt{2}) \end{aligned}$$

(b) Convert the polar equation $r^2 = \frac{\cos\theta}{\sin\theta}$ into a Cartesian equation.

$$\begin{aligned} r^2 &= \frac{\cos\theta}{\sin\theta} \\ r^2 &= \frac{r\cos\theta}{r\sin\theta} \\ x^2 + y^2 &= \frac{x}{y} \end{aligned}$$