Math 251 Name ____KEY____

Quiz 2

You must fully demonstrate your reasoning. An answer without proper justification may receive no credit.

1. Determine if each sequence converges or diverges. If it converges, find the limit of the sequence.

   (a) $\left\{\frac{4n}{3n+2}\right\}_{n=1}^{\infty}$

   $$\lim_{n\to\infty}\frac{4n}{3n+2} \overset{\checkmark}{=} \lim_{n\to\infty}\frac{4}{3} = \frac{4}{3}$$

   Since the limit is a real number,
   the sequence converges (to $\frac{4}{3}$).

   (b) $\left\{\frac{e^n}{n}\right\}_{n=1}^{\infty}$

   $$\lim_{n\to\infty}\frac{e^n}{n} \overset{\checkmark}{=} \lim_{n\to\infty}\frac{e^n}{1} = \infty$$

   Since the limit is not a real number,
   the sequence diverges.

2. Determine if each series converges or diverges. If it converges, find the sum.

   (a) $\sum_{n=1}^{\infty} 2^n$

   This is a geometric series with ratio 2.
   Since the ratio is not between -1 and 1,
   the series diverges, by the Geometric Series Test.

(b) $\sum_{n=1}^{\infty} \frac{1+2^n}{3^n}$

$$\sum_{n=1}^{\infty} \frac{1+2^n}{3^n} = \sum_{n=1}^{\infty} \frac{1}{3^n} + \sum_{n=1}^{\infty} \frac{2^n}{3^n} \quad \text{(OK only if both series converge)}$$

$$= \sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n + \sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n$$

(both ratios are between -1 and 1, so both series converge, by the Geometric Series Test)

$$= \frac{(1)\left(\frac{1}{3}\right)}{1-\left(\frac{1}{3}\right)} + \frac{(1)\left(\frac{2}{3}\right)}{1-\left(\frac{2}{3}\right)}$$

$$= \frac{\left(\frac{1}{3}\right)}{\left(\frac{2}{3}\right)} + \frac{\left(\frac{2}{3}\right)}{\left(\frac{1}{3}\right)}$$

$$= \left(\frac{1}{3}\right)\left(\frac{3}{2}\right) + \left(\frac{2}{3}\right)\left(\frac{3}{1}\right)$$

$$= \frac{1}{2} + 2$$

$$= \frac{5}{2}$$

(c) $\sum_{n=1}^{\infty} \frac{n}{n+1}$

$$\lim_{n\to\infty} \frac{n}{n+1} \overset{\checkmark}{=} \lim_{n\to\infty} \frac{1}{1} = 1$$

Since the limit of the sequence is not 0,

the series $\sum_{n=1}^{\infty} \frac{n}{n+1}$ diverges,

by the Test for Divergence.