Math 152 Name KEY
Quiz 9

If a definite integral is improper, you must evaluate it using the methods discussed in class and in the book.

1. Determine if $\sum_{n=1}^{\infty} \frac{\ln n}{n}$ converges or diverges. You must fully justify your answer.

We will use the Integral Test :

We need to find $\int_1^{\infty} \frac{\ln x}{x} dx$.

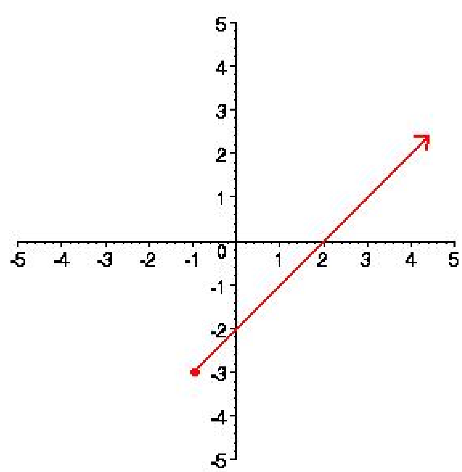
$$\int \frac{\ln x}{x} dx = \int u \, du \qquad (\text{from } u = \ln x, \; du = \frac{1}{x} dx)$$
$$= \frac{1}{2} u^2 + C$$
$$= \frac{1}{2} (\ln x)^2 + C$$

Thus, $$\int_1^{\infty} \frac{\ln x}{x} dx = \lim_{t \to \infty} \int_1^t \frac{\ln x}{x} dx$$
$$= \lim_{t \to \infty} \frac{1}{2} (\ln x)^2 \Big]_1^t$$
$$= \lim_{t \to \infty} \frac{1}{2} (\ln t)^2 - \frac{1}{2}(0) = \infty$$

So the integral diverges.

Therefore, the series also diverges.

2. Let $x = t^2 - 1$, $y = t^2 - 3$, $t \in \mathbb{R}$. Plot a graph of this curve on the $xy-$ axes below.



3. Let $x = 4\sin\theta$, $y = \cos\theta$, $0 \leq \theta \leq \pi$. Plot a graph of the curve on the $xy-$ axes below, and indicate with arrows the direction the curve is traced as θ increases.