1. Fully describe the region of $\mathbb{R}^3$ represented by the equation or inequality:

(a) $9 < x^2 + y^2 + z^2 < 25$

This is the solid region between (but not including) the spheres centered at the origin with radii 3 and 5.

(b) $y > -2$

This is the half-space to the right of (but not including) the vertical plane that is parallel and 2 units to the left of the xz-plane.

(c) $y = x$

This is a vertical plane that makes a 45° angle with both the xz-plane and the yz-plane.

2. Let $\overrightarrow{u} =< -1, 2, 0 >$ and $\overrightarrow{v} =< 4, 1, 1 >$.

(a) Calculate $5\overrightarrow{u} - 3\overrightarrow{v}$.

$$5\overrightarrow{u} - 3\overrightarrow{v} =< -5, 10, 0 > - < 12, 3, 3 >=< -17, 7, -3 >$$

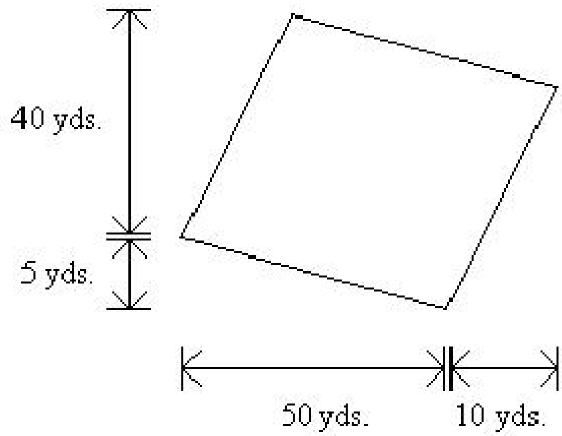
(b) Calculate $\overrightarrow{u} \cdot \overrightarrow{v}$.

$$\overrightarrow{u} \cdot \overrightarrow{v} = (-1)(4) + (2)(1) + (0)(1) = -2$$

(c) Calculate $\overrightarrow{u} \times \overrightarrow{v}$.

$$\overrightarrow{u} \times \overrightarrow{v} = \begin{vmatrix} \overrightarrow{i} & \overrightarrow{j} & \overrightarrow{k} \\ -1 & 2 & 0 \\ 4 & 1 & 1 \end{vmatrix} = 2\overrightarrow{i} + \overrightarrow{j} - 9\overrightarrow{k}$$

3. Suppose that a field is laid out in the parallelogram shape shown below. Find the area of the field.



To do this problem using vectors,

Let $\overrightarrow{a}$ be formed from the edge that tilts down to the right. Let $\overrightarrow{b}$ be formed from the edge that tilts upward to the right.

Then, $\overrightarrow{a} =< 50, -5, 0 >$ and $\overrightarrow{b} =< 10, 40, 0 >$

So, the area of the field is given by

$$A = \left|\overrightarrow{a} \times \overrightarrow{b}\right| = \left|0\,\overrightarrow{i} + 0\,\overrightarrow{j} + 2050\,\overrightarrow{k}\right| = 2,050 \text{ square yards}$$

4. Quick Calculations.

(a) Find the distance from the point $(2, 0, 4)$ to the point $(-1, 2, 2)$.

$$d = \sqrt{(2-(-1))^2 + (0-2)^2 + (4-2)^2} = \sqrt{17}$$

(b) Suppose the angle between the vectors $\overrightarrow{a}$ and $\overrightarrow{b}$ is $60°$, and the magnitudes of $\overrightarrow{a}$ and $\overrightarrow{b}$ are 4 and 5 units, respectively. Find the vector projection of $\overrightarrow{b}$ onto $\overrightarrow{a}$. Express your answer as a scalar multiple of $\overrightarrow{a}$.

$$prog_{\overrightarrow{a}}\,\overrightarrow{b} = \frac{\overrightarrow{a}\cdot\overrightarrow{b}}{|\overrightarrow{a}|^2}\overrightarrow{a} = \frac{|\overrightarrow{a}|\left|\overrightarrow{b}\right|\cos\theta}{|\overrightarrow{a}|^2}\overrightarrow{a}$$

$$= \frac{(4)(5)\cos 60°}{(4)^2}\overrightarrow{a} = \frac{5}{8}\overrightarrow{a}$$

5. Find the equation of the line through the points $(2,1,0)$ and $(1,3,5)$.

$$\overrightarrow{v} =< 1-2, 3-1, 5-0 >=< -1, 2, 5 >$$

$$x = 2 - t,\ \ y = 1 + 2t,\ \ z = 5t$$

6. Find the equation of the plane that is perpendicular to the line with symmetric equations $x = y = 2z$, and contains the line with symmetric equations $2x = 3y = z$.

Note : there is no solution to the problem as stated.
The second line should really be
$(2x = 3y = \frac{-3}{5}z)$ so that it is $\perp$ to the first line.
The work below, however, demonstrates the
method one would use if given perpendicular lines.

$$x = y = 2z \Longrightarrow \frac{x}{2} = \frac{y}{2} = z$$

So, a normal vector to the plane is $< 2, 2, 1 >$.
We can use any point on the second line as a point
on the plane, for example $(1, \frac{2}{3}, 2)$.

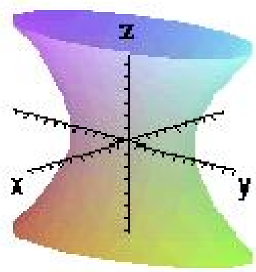
$$2(x-1) + 2(y - \tfrac{2}{3}) + (z-2) = 0$$

7. Find the point at which the plane $2x + y - z = 4$ intersects the line with parametric equations $x = -1 + t, y = 3 - t, z = 4t$.

$$2(-1+t) + (3-t) - (4t) = 4$$
$$-3t = 3$$
$$t = -1$$

$$x = -2, y = 4, z = -4$$
$$(-2, 4, -4)$$

8. Consider the following surface:



Determine which of the following equations describes this surface.

(a) $x^2 + \frac{y^2}{4} + z^2 = 16$

Ellipsoid

(b) $\boxed{x^2 + \frac{y^2}{4} - z^2 = 16}$

Hyperboloid of One Sheet, surrounding z-axis.

(c) $x^2 + \frac{y^2}{4} = 16z^2$

Cone

(d) $x^2 - \frac{y^2}{4} - z^2 = 16$

Hyperboloid of Two Sheet, surrounding x-axis.