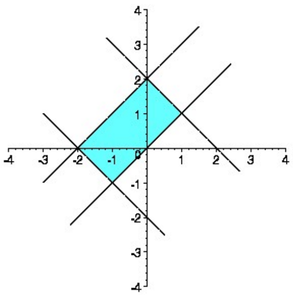
Math 251
Quiz 11
Due Friday, December 7, 2001 at 9:15 am

Name KEY

1. Evaluate $\int\int_R y\; dy\, dx$, where R is the region in the $xy-$plane bounded by the lines $y = x, y = x+2, y = 2-x, y = -2-x$.

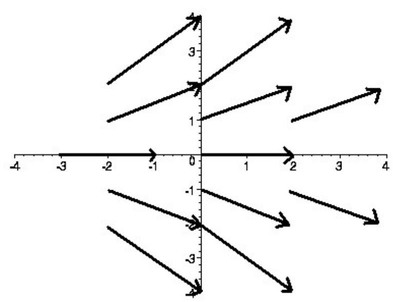


$$\left.\begin{array}{l} u = y - x \\ v = x + y \end{array}\right\} \qquad \begin{array}{l} y = \frac{1}{2}u + \frac{1}{2}v \\ x = \frac{1}{2}v - \frac{1}{2}u \end{array}$$

$$\frac{\partial(x,y)}{\partial(u,v)} = \left| \begin{array}{cc} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{array} \right| = \begin{array}{cc} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{array} = -\frac{1}{2}$$

$$\int\int_R y\; dy\, dx = \int_{-2}^{2}\int_{0}^{2} \left(\frac{1}{2}u + \frac{1}{2}v\right)\left(\frac{1}{2}\right)\, du\, dv = \frac{1}{4}\int_{-2}^{2}\int_{0}^{2} (u+v)\; du\, dv = 2$$

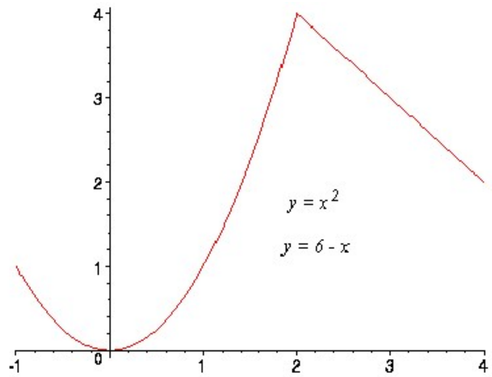
2. (a) Sketch the vector field $\overrightarrow{F}(x, y) =< 2\,,\, y >$.



(b) Show that $\overrightarrow{F}$ is a conservative field, and find a potential function $f(x, y)$.

$$f_x(x, y) = 2 \Longrightarrow f(x, y) = 2x + g(y)$$
$$f_y(x, y) = y \Longrightarrow f(x, y) = \frac{1}{2}y^2 + h(x)$$
$$f(x, y) = 2x + \frac{1}{2}y^2.$$

3. Evaluate $\int_C x \, ds$, where C is the following curve.



$C_1 : x = t, y = t^2, -1 \leq t \leq 2$
$C_2 : x = t, y = 6 - t, 2 \leq t \leq 4$

$$\int_C x ds = \int_{C_1} x ds + \int_{C_2} x ds$$
$$= \int_{-1}^{2} t\sqrt{1^2 + (2t)^2} dt + \int_{2}^{4} t\sqrt{1^2 + (-1)^2} dt$$
$$= \left(\frac{17^{3/2}}{8} - \frac{5^{3/2}}{8}\right) + 6\sqrt{2}$$