Math 251 Name______KEY______
Quiz 2

1. Let $\overrightarrow{a} = < -1, 2, 4 >$ and $\overrightarrow{b} = < -2, -3, 5 >$.

   (a) Find $3\vec{a} - 2\vec{b}$.

$$\begin{aligned} 3\vec{a} - 2\vec{b} &= 3 < -1, 2, 4 > -2 < -2, -3, 5 > \\ &= < -3, 6, 12 > - < -4, -6, 10 > \\ &= < 1, 12, 2 > \end{aligned}$$

   (b) Find $\vec{a} \cdot \vec{b}$.

$$\begin{aligned} \vec{a} \cdot \vec{b} &= < -1, 2, 4 > \cdot < -2, -3, 5 > \\ &= (-1)(-2) + (2)(-3) + (4)(5) \\ &= 2 - 6 + 20 \\ &= 16 \end{aligned}$$

   (c) Find $\vec{a} \cdot \left(\vec{i} + \vec{j}\right)$

$$\begin{aligned} \vec{a} \cdot \left(\vec{i} + \vec{j}\right) &= < -1, 2, 4 > \cdot < 1, 1, 0 > \\ &= (-1)(1) + (2)(1) + (4)(0) \\ &= -1 + 2 + 0 \\ &= 1 \end{aligned}$$

2. Let $\vec{u} = < a, b, \frac{1}{4} >$. Find values of a and b so that $\vec{u}$ is perpendicular to $< 2, 4, 0 >$ and $\vec{u}$ is a unit vector.

   We need to satisfy the following two properties:
   (1) We want $\vec{u} \perp < 2, 4, 0 >$ and (2) We want $|\vec{u}| = 1$.
   In order to satisfy (1), we set $\vec{u} \cdot < 2, 4, 0 > = 0$.

$$\begin{aligned} \vec{u} \cdot < 2, 4, 0 > &= 0 \\ < a, b, \frac{1}{4} > \cdot < 2, 4, 0 > &= 0 \\ 2a + 4b &= 0 \\ 2a &= -4b \\ a &= -2b \end{aligned}$$

In order to satisfy (2), we need to set $|\vec{u}| = 1$.

$$\begin{aligned}
|\vec{u}| &= 1 \\
\sqrt{a^2 + b^2 + \left(\frac{1}{4}\right)^2} &= 1 \\
\sqrt{a^2 + b^2 + \frac{1}{16}} &= 1 \\
a^2 + b^2 + \frac{1}{16} &= 1 \\
a^2 + b^2 &= \frac{15}{16}
\end{aligned}$$

We need to solve the equations $a = -2b$ and $a^2 + b^2 = \frac{15}{16}$ simultaneously.
Substituting the first into the second, we get:

$$\begin{aligned}
(-2b)^2 + b^2 &= \frac{15}{16} \\
4b^2 + b^2 &= \frac{15}{16} \\
5b^2 &= \frac{15}{16} \\
b^2 &= \frac{3}{16} \\
b &= \pm\frac{\sqrt{3}}{4}
\end{aligned}$$

If $b = \frac{\sqrt{3}}{4}$, then $a = \frac{-\sqrt{3}}{2}$. If $b = \frac{-\sqrt{3}}{4}$, then $a = \frac{\sqrt{3}}{2}$.
So there are 2 possible solutions.
$\vec{u} =< \frac{-\sqrt{3}}{2}, \frac{\sqrt{3}}{4}, \frac{1}{4} >$ or $\vec{u} =< \frac{\sqrt{3}}{2}, \frac{-\sqrt{3}}{4}, \frac{1}{4} >$

(Note: you did not need to come up with both solutions.)