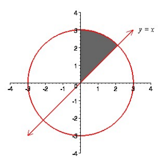
Math 251 Name________KEY________

Quiz 9

1. Evaluate $\int_0^1 \int_{-x}^{x^3} (33xy^2 - 6)\ dydx$.

$$\begin{aligned}
\int_0^1 \int_{-x}^{x^3} (33xy^2 - 6)\ dydx &= \int_0^1 \left.(11xy^3 - 6y)\right]_{y=-x}^{y=x^3}\ dx \\
&= \int_0^1 \left((11x^{10} - 6x^3) - (-11x^4 + 6x)\right)\ dx \\
&= \int_0^1 \left(11x^{10} + 11x^4 - 6x^3 - 6x\right)\ dx \\
&= \left.\left(x^{11} + \frac{11}{5}x^5 - \frac{3}{2}x^4 - 3x^2\right)\right]_{x=0}^{x=1} \\
&= 1 + \frac{11}{5} - \frac{3}{2} - 3 = -\frac{13}{10}
\end{aligned}$$

2. For the shaded region R sketched below,



write out, <u>but do not evaluate</u> an expression with iterated integrals for $\iint_R xy\ dA$, so that:

(a) $dA = dy\,dx$

$$\int_{x=0}^{x=\frac{3}{\sqrt{2}}} \int_{y=x}^{y=\sqrt{9-x^2}} xy\ dy\,dx$$

(b) $dA = dx\,dy$

$$\int_{y=0}^{y=\frac{3}{\sqrt{2}}} \int_{x=0}^{x=y} xy\ dx\,dy + \int_{y=\frac{3}{\sqrt{2}}}^{y=3} \int_{x=0}^{x=\sqrt{9-y^2}} xy\ dx\,dy$$

(c) the integral is expressed solely in polar coordinates.

$$\int_{\theta=\frac{\pi}{4}}^{\theta=\frac{\pi}{2}} \int_{r=0}^{r=3} (r\cos\theta)(r\sin\theta)\ r\ dr\ d\theta$$