Math 381 Final Exam - Take-Home Portion. Name ___________________

This is due at the beginning of the in-class portion of the final exam, i.e., at 10:30 am on Wednesday, December 11, 2002. Write out your solutions on your own paper and then staple your solutions together in order with this page as the cover page. The take-home portion is worth 30% of the total final exam score.

You may use any theorems, corollaries, definitions, etc. from the book in your proofs. You may not use any exercises from the book in your proofs unless instructed otherwise. You may use only the book and your class notes. You may not work with or get help from anyone else.

1. Let $a, b, c \in \mathbb{R}$, with $0 \leq c \leq a$. Prove directly from the Field and Order Properties that if $ab < c$ then $b < 1$.

2. Let x be irrational. Prove that there exists a sequence (x_n) in $\mathbb{Q}$ with $\lim x_n = x$. You may use Exercise 8.5(a).

3. Let (s_n) and (t_n) be nonempty bounded sequences of real numbers.

   (a) Prove: For all $N \in \mathbb{N}$, $\inf \{s_n + t_n : n > N\} \geq \inf \{s_n : n > N\} + \inf \{t_n : n > N\}$.

   (b) Prove: $\liminf (s_n + t_n) \geq \liminf s_n + \liminf t_n$. You may use part (a) and Exercise 9.9(c) from the book.

   (c) Give an example of a pair of sequences (s_n) and (t_n) such that $\liminf (s_n + t_n) > \liminf s_n + \liminf t_n$

4. Let $a, b \in \mathbb{R}$. Let f be a continuous function on $[a, b]$ such that for all $x \in [a, b]$, $f(x) \neq 0$. Prove that for all $y, z \in (a, b)$, $f(y)f(z) > 0$.