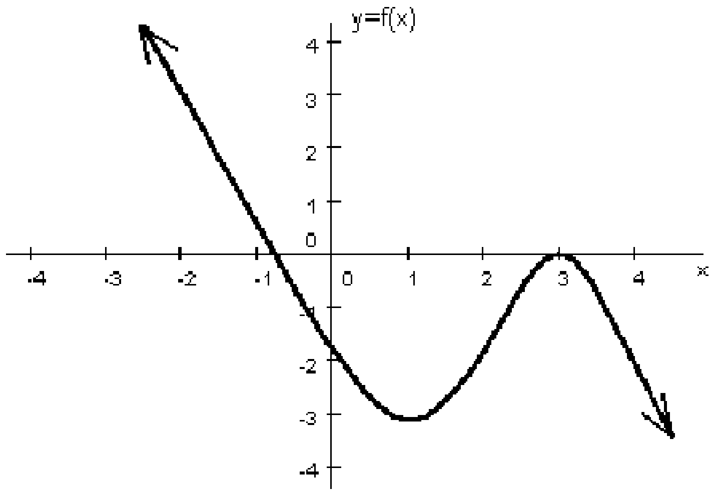
1. Let $f(x)$ be given by the following graph:



(a) Give a careful estimate of $(f \circ f)(3)$

$$(f \circ f)(3) = f(f(3)) = f(0) \approx -1.8$$

(b) Give a careful estimate of the average rate of change of $f(x)$ from $x = 1$ to $x = 3$.

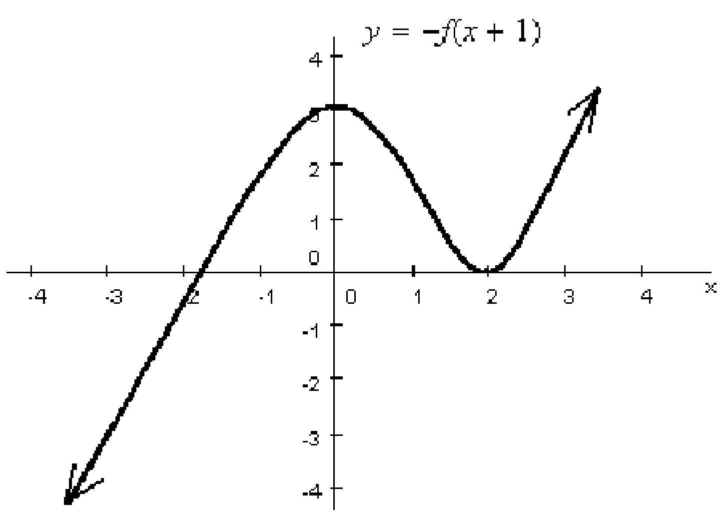
$$\text{Average R.O.C.} = \frac{f(3) - f(1)}{3 - 1} \approx \frac{0 - (-3)}{2} = \frac{3}{2}$$

(c) Give a careful estimate of $f'(1)$

$f'(1) \approx 0$ (since the curve is approximately horizontal there.)

(d) Carefully graph $y = -f(x+1)$ on the axes below.

The "+1" shifts the graph to the left by one and the minus out front reflects the graph over the $x-$axis.



2. Find the indicated limit. If you find that the limit does not exist, you must describe the limit as ∞ or $-\infty$ if either of these descriptions is appropriate.

(a) $\lim\limits_{x \to 3} \dfrac{9-x^2}{x-3}$

$$\lim_{x \to 3} \frac{\overbrace{9-x^2}^{\overset{0}{\uparrow}}}{\underbrace{x-3}_{\underset{0}{\downarrow}}} = \lim_{x \to 3} \frac{(3-x)(3+x)}{x-3} = \lim_{x \to 3} -(3+x) = -6$$

(b) $\lim\limits_{x \to 3} \dfrac{x-7}{(x-3)^2}$

$$\lim_{x \to 3} \frac{\overbrace{x-7}^{\overset{-4}{\uparrow}}}{\underbrace{(x-3)^2}_{\underset{0}{\downarrow}}}$$ We will consider the left and righthand limits separately.

$$\lim_{x \to 3^+} \frac{\overbrace{x-7}^{\overset{-4}{\uparrow}}}{\underbrace{(x-3)^2}_{\underset{0,\text{ but }+}{\downarrow}}} = -\infty$$

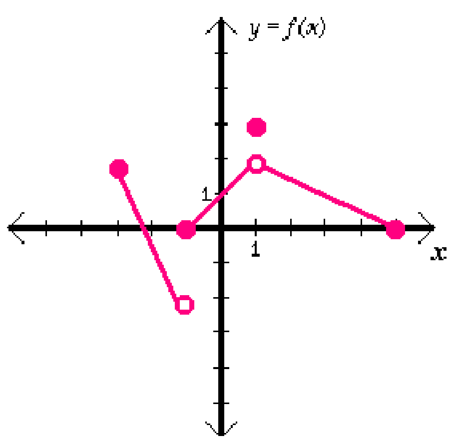
$$\lim_{x \to 3^-} \frac{\overbrace{x-7}^{\overset{-4}{\uparrow}}}{\underbrace{(x-3)^2}_{\underset{0,\text{ but }+}{\downarrow}}} = -\infty$$

Thus, $\lim\limits_{x \to 3} \dfrac{x-7}{(x-3)^2} = -\infty$

3. Sketch a graph on the axes provided of a function that satisfies the stated conditions.

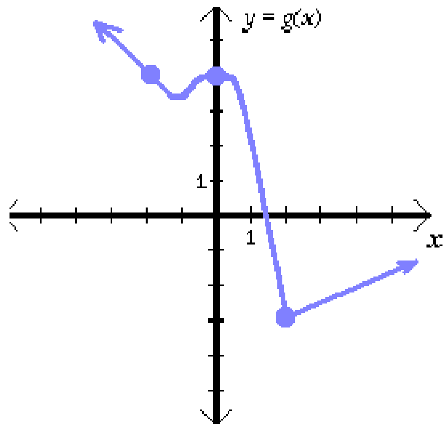
(a) A function $f(x)$ with domain $= [-2, 5]$, such that $f(-1) = 0, f(1) = 3, \lim\limits_{x \to -1} f(x)$ DNE, and $\lim\limits_{x \to 1} f(x) = 2$.

There are many possible solutions. Here is one:



(b) A function $g(x)$ such that $g(-2) = 4, g(0) = 4, g(2) = -3, g'(-2) = -1, g'(0) = 0, g'(2)$ DNE.

Here is one possible solution, of many.



4. Let $f(x) = x^3 - 4x + 5$.

(a) Find $f'(2)$.

$$\begin{aligned} f'(x) &= 3x^2 - 4 \\ f'(2) &= 3(2)^2 - 4 = 8 \end{aligned}$$

(b) Find the equation of the tangent line to the function at the point where $x = 2$.

$$\begin{aligned} \text{When } x &= 2, \text{ we have } y = f(2) = 2^3 - 4(2) + 5 = 5 \\ y - y_0 &= m(x - x_0) \\ y - 5 &= 8(x - 2) \\ y &= 8x - 11 \end{aligned}$$

5. For each of the following curves, find $\dfrac{dy}{dx}$. Do not simplify your final answers.

(a) $y = \sqrt[3]{x} - \dfrac{2}{\sqrt{x}} - 5\cos x$

$$\begin{aligned} y &= x^{1/3} - 2x^{-1/2} - 5\cos x \\ \frac{dy}{dx} &= \frac{1}{3}x^{-2/3} + x^{-3/2} + 5\sin x \\ &= \frac{1}{3\sqrt[3]{x^2}} + \frac{1}{\sqrt{x^3}} + 5\sin x \end{aligned}$$

(b) $y = \dfrac{x^2 - 3x + 7}{x^4 + 2x}$

$$\begin{aligned} \frac{dy}{dx} &= \frac{(x^4 + 2x)\frac{d}{dx}(x^2 - 3x + 7) - (x^2 - 3x + 7)\frac{d}{dx}(x^4 + 2x)}{(x^4 + 2x)^2} \\ &= \frac{(x^4 + 2x)(2x - 3) - (x^2 - 3x + 7)(4x^3 + 2)}{(x^4 + 2x)^2} \end{aligned}$$

6. (a) The limit expression $\displaystyle\lim_{h \to 0} \frac{\sin\left(\frac{\pi}{6} + h\right) - \frac{1}{2}}{h}$ is equal to the instantaneous rate of change of a particular function $f(x)$ at the number a. What are $f(x)$ and a?

$f'(a) = \displaystyle\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

Matching this expression with the one in the problem, we see

$a = \dfrac{\pi}{6}$ and $f(x) = \sin x$

(b) Let $w(x) = x^3 g(x)$. If $g(1) = -2$, and $g'(1) = 4$, find $w'(1)$.

$$\begin{aligned} w'(x) &= x^3 g'(x) + 3x^2 g(x) \\ w'(1) &= 1^3 g'(1) + 3(1)^2 g(1) \\ &= (1)(4) + (3)(-2) \\ &= -2 \end{aligned}$$