Math 381 Final Exam - Take-Home Portion. Name ____________________

This is due at the beginning of the in-class portion of the final exam, i.e., at 10:30 am on Wednesday, December 10, 2003. Write out your solutions on your own paper and then staple your solutions together (in order) with this page as the cover page. The take-home portion is worth 20% of the total final exam score.

You may use any theorems, corollaries, definitions, etc. from the book in your proofs. You may not use any exercises from the book in your proofs unless instructed otherwise. You may use only the book and your class notes. You may ask me questions of a general nature that I may or may not choose to answer. You may not work with or get help from anyone else.

1. Let $a, c \in \mathbb{R}$, with $0 \leq c \leq a$. Prove directly from the Field and Order Properties that if $a^2 < c$ then $a < 1$.

2. Let $a, b \in \mathbb{R}$. Let f be a continuous function on $[a, b]$ such that for all $x \in [a, b]$, $f(x) \neq 0$. Prove that for all $y, z \in (a, b)$, $f(y)f(z) > 0$.

3. Def: Let f be a real-valued function whose domain is a subset of $\mathbb{R}$. The function f is *mellow at* $a \in \text{dom } f$ if, $\forall \varepsilon > 0, \exists \beta > 0$ such that $\forall x \in \text{dom } f$ with $|x - a| < \beta$, we have $|f(x) - f(a)| < \varepsilon + 1$.

   (a) Give an example of a function that is mellow at 2 but not continuous at 2.

   (b) Prove: If f is continuous at a then f is mellow at a.