

Math 152 - Lab 2. Alternating Series and Finding Formulas for Terms. This lab is due Wednesday, September 15.

Alternating sequences and series are sequences and series with terms that have $(-1)^n$, $(-1)^{n+1}$, or a similar expression in them.

1. Find the first 10 terms of each sequence.

(a) $a_n = \frac{(-1)^{n+1}}{n}$

(b) $a_n = \frac{(-1)^{n-1}}{n^2}$

(c) $a_n = \frac{(-1)^n n}{n+1}$

2. Why are sequences of this type called ‘alternating’ sequences?

3. Complete the following chart.

a_n	$\lim_{n \rightarrow \infty} a_n$	$\sum_{n=1}^{n=10} a_n$	$\sum_{n=1}^{n=1000} a_n$	$\sum_{n=1}^{n=10,000} a_n$	Do you think $\sum_{n=1}^{n=\infty} a_n$ converges or diverges?
$\frac{(-1)^{n+1}}{n}$					
$\frac{(-1)^{n+1}}{n^2}$					
$\frac{(-1)^n n}{n+1}$					
$\frac{(-1)^n}{n^{1/2}}$					
$\frac{(-1)^{n-1} n}{n^2 + 1}$					
$\frac{(-1)^n n^2}{5n^2 + 3}$					

4. Make a reasonable conjecture based on this evidence about how $\lim_{n \rightarrow \infty} a_n$ is related to the convergence of $\sum_{n=1}^{n=\infty} a_n$ when a_n is an alternating sequence. (Warning: The conjecture you will make based on this evidence will be slightly incorrect. We will discuss the correct theorem in class.)

5. Consider the sequence $\{3n + 5\}_{n \geq 1}$

(a) State the first five terms.

(b) Is the difference between consecutive terms constant? If so, what is the difference?

6. Consider the sequence $\{5n + 3\}_{n \geq 1}$.

(a) State the first five terms.

(b) Is the difference between consecutive terms constant? If so, what is the difference?

7. Consider the sequence $\{-2n + 1\}_{n \geq 1}$.

(a) State the first five terms.

(b) Is the difference between consecutive terms constant? If so, what is the difference?

8. Based on the results of problems 1-3, complete the following statements.

i) The sequence defined by $a_n = dn + c$, for $n \geq 1$, begins with the term _____ and the difference between consecutive terms is _____.

ii) A sequence that begins with the term m and has a difference of t between consecutive terms can be defined using the formula $a_n = \underline{\hspace{2cm}}$, for $n \geq 1$.

Sequences of the form $a_n = dn + c$ are called **Arithmetic Sequences**.

9. For each of the sequences below, give the formula for a_n .

(a) $\{3, 7, 11, 15, 19, 23, \dots\}$

(b) $\{-2, -6, -10, -14, -18, -22, \dots\}$

(c) The sequence of odd positive numbers beginning with 1.

(d) The sequence of even positive numbers beginning with 2.

(e) The sequence of even nonnegative numbers beginning with 0.

10. Consider the sequence $\{3(2)^n\}_{n \geq 1}$.

(a) What is the name for this type of sequence?

(b) State the first 5 terms.

(c) Is the ratio between consecutive terms constant? If so, what number must you multiply each term by to get the term that follows it?

11. Consider the sequence $\left\{2\left(\frac{3}{4}\right)^n\right\}_{n \geq 1}$.

(a) State the first 5 terms.

(b) Is the ratio between consecutive terms constant? If so, what number must you multiply each term by to get the term that follows it?

12. Consider the sequence $\left\{\frac{-5}{3^n}\right\}_{n \geq 1}$.

(a) State the first 5 terms.

(b) Is the ratio between consecutive terms constant? If so, what number must you multiply each term by to get the term that follows it?

13. Consider the sequence $\left\{\frac{2(-5)^n}{3^n}\right\}_{n \geq 1}$.

(a) State the first 5 terms.

(b) Is the ratio between consecutive terms constant? If so, what number must you multiply each term by to get the term that follows it?

14. Based on the results of problems 6-9, complete the following statements.

i) The sequence defined by $a_n = c(r)^n$, for $n \geq 1$, begins with the term _____ and the ratio of consecutive terms is _____.

ii) A sequence that begins with the term m and has a ratio of t from each term to its predecessor can be defined using the formula $a_n = \underline{\hspace{2cm}}$, for $n \geq 1$.

15. For each of the sequences below, give the formula for a_n so that the first term is a_1 .

(a) $\{3, 12, 48, 192, 768, \dots\}$

(b) $\{10, 100, 1000, 10000, \dots\}$

(c) $\{2, 4, 8, 16, 32, 64, \dots\}$

(d) $\{1, 2, 4, 8, 16, 32, 64, \dots\}$

(e) $\left\{ \frac{1}{5}, \frac{2}{25}, \frac{4}{125}, \frac{8}{625}, \frac{16}{3125}, \dots \right\}$

(f) $\left\{ \frac{9}{10}, \frac{-27}{100}, \frac{81}{1000}, \frac{-243}{10000}, \frac{729}{100,000}, \dots \right\}$

16. Most sequences are neither arithmetic nor geometric, but you can derive the formula for a_n by using what you have learned about arithmetic and geometric sequences. For each of the sequences below, give the formula for a_n so that the first term is a_1 .

(a) $\left\{ \frac{2}{5}, \frac{4}{25}, \frac{6}{125}, \frac{8}{625}, \frac{10}{3125}, \dots \right\}$ (Hint: Start by writing the formula for the numerator and denominator separately.)

(b) $\left\{ \frac{3}{5}, \frac{-3}{7}, \frac{1}{3}, \frac{-3}{11}, \frac{3}{13}, \dots \right\}$

(c) $\left\{ \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \dots \right\}$

(d) $\left\{ \frac{1}{4}, \frac{2}{9}, \frac{3}{16}, \frac{4}{25}, \frac{5}{36}, \dots \right\}$