

1. Evaluate $\int \frac{x^3}{x+1} dx$

$$\begin{array}{r}
 x+1 \overline{)x^3} \\
 \underline{x^3 + x^2} \\
 -x^2 \\
 \underline{-x^2 - x} \\
 x \\
 \underline{x+1} \\
 -1
 \end{array}$$

$$\begin{aligned}
 \int \frac{x^3}{x+1} dx &= \int \left(x^2 - x + 1 - \frac{1}{x+1} \right) dx \\
 &= \frac{1}{3}x^3 - \frac{1}{2}x^2 + x - \ln|x+1| + C
 \end{aligned}$$

2. Find the partial fraction decomposition of the rational function $\frac{4x}{(x-3)(x+1)}$

$$\begin{aligned}
 \frac{4x}{(x-3)(x+1)} &= \frac{A}{x-3} + \frac{B}{x+1} \\
 4x &= A(x+1) + B(x-3)
 \end{aligned}$$

$$x = -1 :$$

$$-4 = -4B \implies B = 1$$

$$x = 3 :$$

$$12 = 4A \implies A = 3$$

$$\frac{4x}{(x-3)(x+1)} = \frac{3}{x-3} + \frac{1}{x+1}$$

3. Gilly, a student in Math 152 has completed the partial fraction decomposition of a rational function integrand and arrived at the following integral:

$$\int 2 - \frac{3}{x-4} + \frac{2}{x^2+9} + \frac{x}{x^2+9} dx$$

Finish this problem for Gilly by evaluating this integral.

$$\int \left(2 - \frac{3}{x-4} + \frac{2}{x^2+9} + \frac{x}{x^2+9} \right) dx = 2x - 3 \ln|x-4| + \frac{2}{3} \arctan\left(\frac{x}{3}\right) + \int \frac{x}{x^2+9} dx$$

$$\begin{aligned} u &= x^2 + 9 \\ \frac{du}{dx} &= 2x \\ dx &= \frac{du}{2x} \end{aligned}$$

$$\begin{aligned} \int \frac{x}{x^2+9} dx &= \frac{1}{2} \int \frac{1}{u} du \\ &= \frac{1}{2} \ln|u| + C \\ &= \frac{1}{2} \ln|x^2+9| + C \end{aligned}$$

$$\int \left(2 - \frac{3}{x-4} + \frac{2}{x^2+9} + \frac{x}{x^2+9} \right) dx = 2x - 3 \ln|x-4| + \frac{2}{3} \arctan\left(\frac{x}{3}\right) + \frac{1}{2} \ln|x^2+9| + C$$