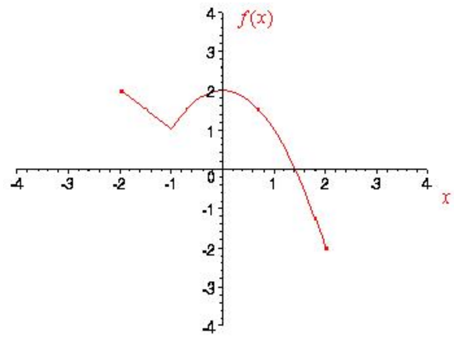
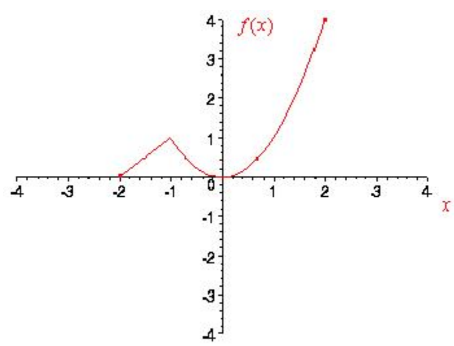
1. Consider the function $f(x)$ drawn below.



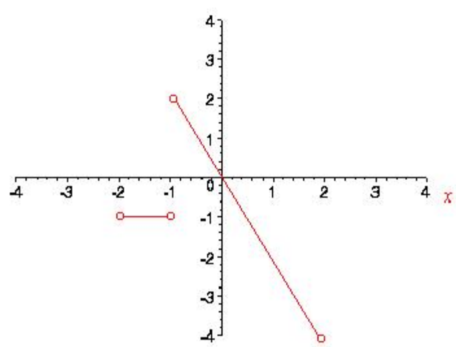
(a) State the domain of $f(x)$.

$$[-2, 2]$$

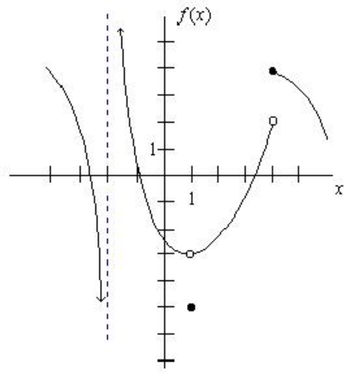
(b) Sketch a graph of $y = 2 - f(x)$ on the axes below.



(c) Sketch a graph of $f'(x)$ on the axes below.



2. Consider the graph of $f(x)$ drawn below.



(a) Find (*i*) $\lim_{x\to -2^-} f(x)$, (*ii*) $\lim_{x\to -2^+} f(x)$, and (*iii*) $\lim_{x\to -2} f(x)$

$\lim_{x\to -2^-} f(x) = -\infty$ $\quad$ $\lim_{x\to -2^+} f(x) = \infty$ $\quad$ $\lim_{x\to -2} f(x)$ does not exist

(b) Find (*i*) $\lim_{x\to 1^-} f(x)$, (*ii*) $\lim_{x\to 1^+} f(x)$, and (*iii*) $\lim_{x\to 1} f(x)$

$\lim_{x\to 1^-} f(x) = -3$ $\quad$ $\lim_{x\to 1^+} f(x) = -3$ $\quad$ $\lim_{x\to 1} f(x) = -3$

(c) Find (*i*) $\lim_{x\to 4^-} f(x)$, (*ii*) $\lim_{x\to 4^+} f(x)$, and (*iii*) $\lim_{x\to 4} f(x)$

$\lim_{x\to 4^-} f(x) = 2$ $\quad$ $\lim_{x\to 4^+} f(x) = 4$ $\quad$ $\lim_{x\to 4} f(x)$ does not exist

3. (a) Find the average rate of change of the function $\cos x$ over the interval $[0, \pi]$.

$$\text{average rate of change} = \frac{f(\pi) - f(0)}{\pi - 0} = \frac{-1-1}{\pi} = -\frac{2}{\pi}$$

(b) Let $f(x) = x^2$. Find $f'(x)$ using the formal (limit) definition of the derivative.

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h} \\ &= \lim_{h\to 0} \frac{x^2+2xh+h^2-x^2}{h} \\ &= \lim_{h\to 0} \frac{2xh+h^2}{h} \\ &= \lim_{h\to 0} 2x+h \\ &= 2x \end{aligned}$$

OR

$$\begin{aligned} f'(x) &= \lim_{b\to x} \frac{f(b)-f(x)}{b-x} \\ &= \lim_{b\to x} \frac{b^2-x^2}{b-x} \\ &= \lim_{b\to x} \frac{(b-x)(b+x)}{b-x} \\ &= \lim_{b\to x} b+x \\ &= x+x \\ &= 2x \end{aligned}$$

4. (a) Let $f(x) = \sqrt[3]{x} \sin x$. Find $f'(x)$.

$$\begin{aligned} f(x) &= x^{1/3} \cdot \sin x \\ f'(x) &= \frac{1}{3}x^{-2/3} \sin x + x^{1/3} \cos x \\ &= \frac{\sin x}{3\sqrt[3]{x^2}} + \sqrt[3]{x} \cos x \end{aligned}$$

(b) Let $f(x) = \frac{x^2+9x+3}{x^2+4}$. Find $f'(x)$.

$$f'(x) = \frac{\left(x^2+4\right)(2x+9) - \left(x^2+9x+3\right)(2x)}{\left(x^2+4\right)^2}$$

5. Let $f(x) = \left(3x^2 - 6x + 1\right)^{10}$. Find the equation of the tangent line to $f(x)$ at the point where $x = 0$.

$$\begin{aligned} f'(x) &= 10\left(3x^2 - 6x + 1\right)^9 (6x - 6) \\ m &= f'(0) = 10(1)(-6) = -60 \\ (x_0, y_0) &= (0, 1) \end{aligned}$$

So the tangent line is:

$$\begin{aligned} y - 1 &= -60(x - 0) \\ y &= -60x + 1 \end{aligned}$$

6. (a) Suppose that $f(x) = g(x)h(x) - g(x)$ and $g(2) = 3, h(2) = 5, g'(2) = 4$, and $h'(2) = 6$. Find $f'(2)$.

$$\begin{aligned} f'(x) &= g'(x)h(x) + g(x)h'(x) - g'(x) \\ f'(2) &= g'(2)h(2) + g(2)h'(2) - g'(2) \\ &= (4)(5) + (3)(6) - 4 \\ &= 34 \end{aligned}$$

(b) Let $h(x) = f(x^2)$. Find $h'(x)$.

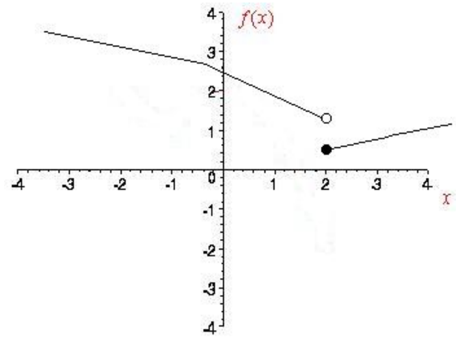
$$h'(x) = f'\left(x^2\right) \cdot 2x$$

7. (a) Give an example of a function $f(x)$ that is continuous for all real values of x, but is not differentiable at $x = 0$.

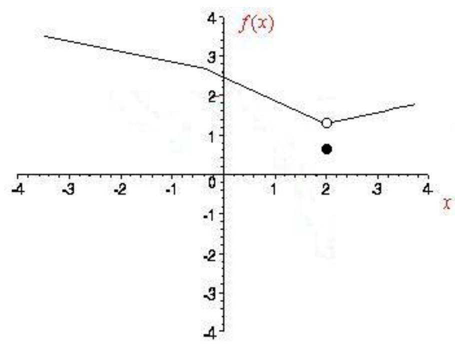
One example is $f(x) = |x|$. Another is $f(x) = \sqrt[3]{x}$.

(b) Sketch the graph of a function $f(x)$ that is defined at $x = 2$, but is not continuous there.

One possible graph is



Another is



(c) If $f(x) = 3x + 4$ and $g(x) = x^2 - 4x$, find $f \circ g(x)$ and $g \circ f(x)$. Clearly label each.

$$f \circ g(x) = f(g(x)) = f\left(x^2 - 4x\right) = 3\left(x^2 - 4x\right) + 4 = 3x^2 - 12x + 4$$

$$\begin{aligned} g \circ f(x) = g(f(x)) = g(3x+4) &= (3x+4)^2 - 4(3x+4) \\ &= 9x^2 + 24x + 16 - 12x - 16 \\ &= 9x^2 + 12x \end{aligned}$$