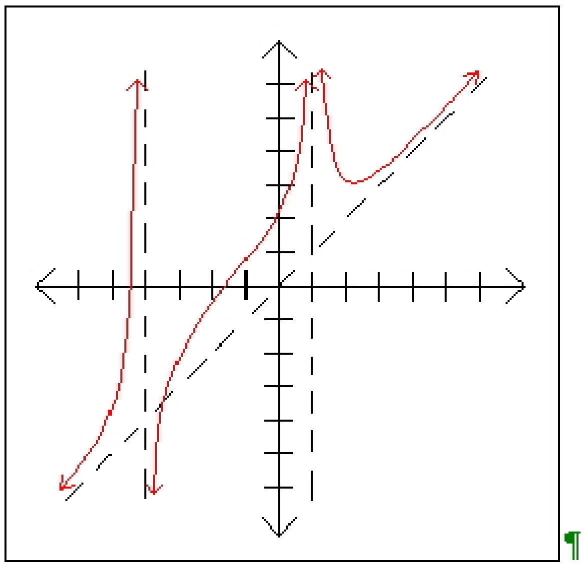
1. Sketch the graph of a rational function $f(x)$ with the following properties:

   i. There is a slant asymptote of $y = x$ and vertical asymptotes of $x = -4$ and $x = 1$.

   ii. $f'(x) > 0$ everywhere except for $1 < x < 2$.

   iii. $f''(x) > 0$ everywhere except for $-4 < x < -1$.



2. Suppose that $f''(x) = x + \sqrt{x}$, $f(1) = 1$, and $f'(1) = 2$. Find $f(x)$.

$$f''(x) = x + x^{1/2}$$
$$f'(x) = \frac{1}{2}x^2 + \frac{2}{3}x^{3/2} + C$$
$$f'(1) = \frac{1}{2} + \frac{2}{3} + C \text{ and } f'(1) = 2, \text{ so } C = 2 - \frac{1}{2} - \frac{2}{3} = \frac{5}{6}$$
$$f'(x) = \frac{1}{2}x^2 + \frac{2}{3}x^{3/2} + \frac{5}{6}$$
$$f(x) = \frac{1}{6}x^3 + \frac{4}{15}x^{5/2} + \frac{5}{6}x + D$$
$$f(1) = \frac{1}{6} + \frac{4}{15} + \frac{5}{6} + D \text{ and } f(1) = 1, \text{ so } D = 1 - \frac{1}{6} - \frac{4}{15} - \frac{5}{6} = -\frac{4}{15}$$
$$f(x) = \frac{1}{6}x^3 + \frac{4}{15}x^{5/2} + \frac{5}{6}x - \frac{4}{15}$$

3. Find any and all asymptotes (vertical, horizontal, slant) for each function.

(a) $f(x) = \frac{3x^2+1}{x^2-4}$

Vertical asymptotes are $x = 2$ and $x = -2$.

$$\lim_{x\to\infty} \frac{3x^2+1}{x^2-4} = \lim_{x\to\infty} \frac{x^2\left(3+\frac{1}{x^2}\right)}{x^2\left(1-\frac{4}{x^2}\right)} = \lim_{x\to\infty} \frac{3+\frac{1}{x^2}}{1-\frac{4}{x^2}} = \frac{3}{1} = 3$$

$$\lim_{x\to-\infty} \frac{3x^2+1}{x^2-4} = \lim_{x\to-\infty} \frac{x^2\left(3+\frac{1}{x^2}\right)}{x^2\left(1-\frac{4}{x^2}\right)} = \lim_{x\to-\infty} \frac{3+\frac{1}{x^2}}{1-\frac{4}{x^2}} = \frac{3}{1} = 3$$

So $y = 3$ is a horizontal asymptote.

(b) $g(x) = \frac{x^2+6x}{x+1}$

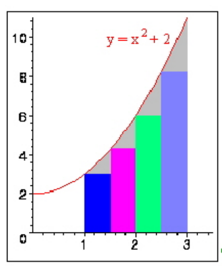
Vertical asymptote is $x = -1$.

Because the numerator has degree 2 and the denominator has degree 1 (1 less than 2), this function has a slant asymptote.

$$\begin{array}{r l} & x+5 \\ x+1 \,\big) & x^2+6x \\ & \underline{x^2+x} \\ & \quad 5x \\ & \quad \underline{5x+5} \\ & \qquad -5 \end{array}$$

The slant asymptote is $y = x + 5$.

4. Sketch the region beneath the curve $f(x) = x^2 + 2$ and above the $x-$axis from $x = 1$ to $x = 3$. Then estimate the area of this region by using 4 rectangles and left endpoints (that is, find L_4).



$$A_1 = (1^2+2)\left(\frac{1}{2}\right) = \frac{3}{2}$$

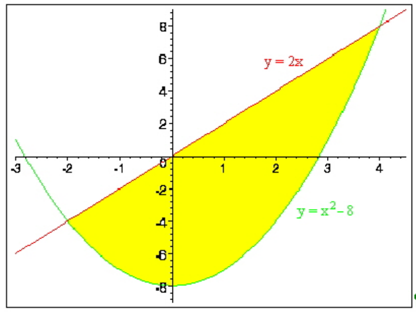
$$A_2 = \left(\left(\frac{3}{2}\right)^2+2\right)\left(\frac{1}{2}\right) = \frac{17}{8}$$

$$A_2 = (2^2+2)\left(\frac{1}{2}\right) = 3$$

$$A_2 = \left(\left(\frac{5}{2}\right)^2+2\right)\left(\frac{1}{2}\right) = \frac{33}{8}$$

$$L_4 = \frac{3}{2} + \frac{17}{8} + 3 + \frac{33}{8} = \frac{41}{4}$$

5. (a) Sketch the region enclosed by the curves $y = 2x$, and $y = x^2 - 8$.



(b) Write a definite integral that represents the area of the region in part (a).

$$\int_{-2}^{4} 2x - \left(x^2 - 8\right)\, dx = \int_{-2}^{4} 2x - x^2 + 8\, dx$$

6. Evaluate each integral.

(a) $\int \frac{2}{x^3} + 5\ dx$

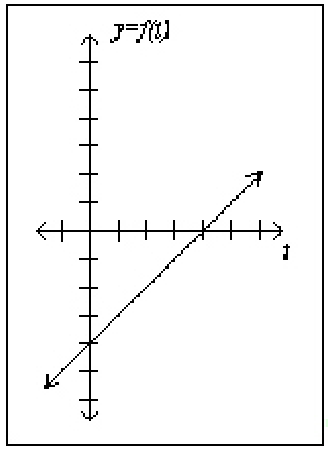
$$\begin{aligned}\int \frac{2}{x^3} + 5\ dx &= \int 2x^{-3} + 5\ dx \\ &= -x^{-2} + 5x + C \\ &= -\frac{1}{x^2} + 5x + C\end{aligned}$$

(b) $\int_0^2 5x^2\sqrt{x^3+1}\ dx$

$$\begin{aligned}u &= x^3 + 1 \\ du &= 3x^2\, dx\end{aligned}$$

$$\begin{aligned}\int_0^2 5x^2\sqrt{x^3+1}\ dx &= \int_1^9 \frac{5}{3}\sqrt{u}\ du \\ &= \int_1^9 \frac{5}{3}u^{1/2}\ du \\ &= \frac{10}{9}u^{3/2}\Big]_1^9 \\ &= 30 - \frac{10}{9} = \frac{260}{9}\end{aligned}$$

7. Let $f(t)$ be the function graphed below.



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(a) Find $\int_1^5 f(t)\ dt$

$$\frac{1}{2}(1)(1)-\frac{1}{2}(3)(3)=-4$$

(b) Find $\int_1^5 |f(t)|\ dt$

$$\frac{1}{2}(1)(1)+\frac{1}{2}(3)(3)=5$$

8. Short answer.

(a) If oil leaks from a tank at a rate of $r(t)$ gallons per minute after t minutes, state clearly what $\int_0^{200} r(t)\ dt$ represents.

The total amount of oil (in gallons) that leaked during the first 200 minutes.

(b) Let $f(x) = x^3 + \sqrt{x}$. Find an expression for the area under the grah of $f(x)$ from $x = 3$ to $x = 9$ as a limit (of a sum).

$$\lim_{n\to\infty}\sum_{i=0}^{n}\left(\left(3+\frac{6i}{n}\right)^3+\sqrt{3+\frac{6i}{n}}\right)\left(\frac{6}{n}\right)$$

(c) Let $g(x) = \int_{x^2}^{\pi} \cos\left(5t^3\right)\ dt$. Find $g'(x)$.

$$\begin{aligned} g'(x) &= \cos\left(5\left(x^2\right)^3\right)\cdot 2x \\ &= 2x\cos\left(5x^6\right) \end{aligned}$$