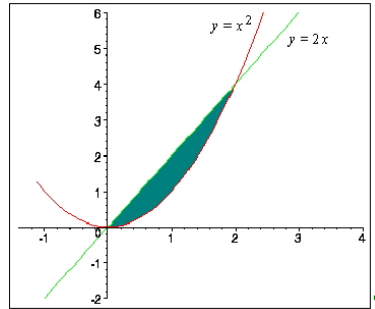


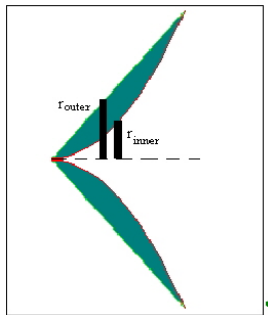
1. Find the average value of the function $f(x) = x^2 + 1$ over the interval $[1, 4]$.

$$f_{ave} = \frac{1}{4-1} \int_1^4 x^2 + 1 \, dx = \frac{1}{3} \int_1^4 x^2 + 1 \, dx = \frac{1}{3} \left[\frac{1}{3}x^3 + x \right]_1^4 = \frac{1}{3} \left(\left(\frac{64}{3} + 4 \right) - \left(\frac{1}{3} + 1 \right) \right) = 8$$

2. Write a definite integral expression that represents the volume of the solid obtained by rotating the following region about the indicated axis:



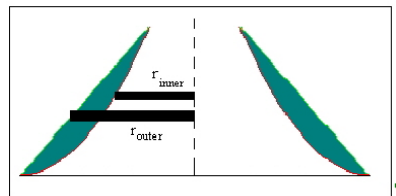
- (a) The x -axis.



$$r_{outer} = 2x, \quad r_{inner} = x^2$$

$$V = \pi \int_0^2 (2x)^2 - (x^2)^2 \, dx = \pi \int_0^2 4x^2 - x^4 \, dx$$

- (b) The axis $x = 3$.



$$r_{outer} = 3 - 2x, \quad r_{inner} = 3 - x^2$$

$$V = \pi \int_0^2 (3 - 2x)^2 - (3 - x^2)^2 \, dx = \pi \int_0^2 10x^2 - 12x - x^4 \, dx$$