

Quiz 5

1. Let $f(x) = \frac{x}{x+3}$.

(a) Find $f''(x)$.

$$\begin{aligned} f'(x) &= \frac{(x+3)(1) - (x)(1)}{(x+3)^2} \\ &= \frac{x+3-x}{(x+3)^2} \\ &= \frac{3}{(x+3)^2} \end{aligned}$$

$$\begin{aligned} f''(x) &= \frac{(x+3)^2(0) - (3)(2(x+3)(1))}{((x+3)^2)^2} \\ &= \frac{-6(x+3)}{(x+3)^4} \\ &= -\frac{6}{(x+3)^3} \end{aligned}$$

(b) Find $f''(0)$.

$$f''(0) = -\frac{6}{(0+3)^3} = -\frac{6}{27} = -\frac{2}{9}$$

2. Find the linearization of $f(x) = \sqrt{x}$ at $a = 4$.

$$\begin{aligned} f'(x) &= \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}} \\ f'(4) &= \frac{1}{2\sqrt{4}} = \frac{1}{4} \\ \text{and } f(4) &= 2 \\ \text{So, } L(x) &= \frac{1}{4}(x-4) + 2 \end{aligned}$$

3. Let $y = x^2 + 1$. Let $x = 2$, and $dx = 0.1$

See the picture on the next page to see how all of the different values relate to the graph.

(a) Find Δy .

$$\begin{aligned} \Delta y &= f(2.1) - f(2) \\ &= ((2.1)^2 + 1) - (2^2 + 1) \\ &= 5.41 - 5 \\ &= .41 \end{aligned}$$

(b) Find dy .

$$\begin{aligned} dy &= f'(x)dx \\ &= 2x dx \\ &= 2(2)(.1) \\ &= .4 \end{aligned}$$

OR

$$\begin{aligned} dy &= L(2.1) - f(2), \text{ where } L(x) = 4(x - 2) + 5 \text{ from linearization formula.} \\ &= (4(2.1 - 2) + 5) - 5 \\ &= .4 \end{aligned}$$

Note: I have purposely miscaled this graph so that the quantities would be easier to read.

