Math 361 Name Key

Quiz1

1. Prove: If $A \subseteq C$ and $B \subseteq C$ then $A \triangle B \subseteq C$. (Don't forget to state your premises and your conclusion.)

   Note: The following is only one possible proof out of many.

   Proof. Premises: $A \subseteq B$ and $B \subseteq C$.

   Let x be an arbitrarily chosen element of $A \triangle B$.

   Then either $x \in A$ and $x \notin B$ or $x \notin A$ and $x \in B$.

   If $x \in A$ and $x \notin B$ then $x \in C$, since $A \subseteq C$.

   If $x \notin A$ and $x \in B$ then $x \in C$, since $B \subseteq C$.

   In either case, $x \in C$.

   Conclusion: $A \triangle B \subseteq C$. ■

2. Determine if the given operation $*$ on the set S is a binary operation. **If you determine that it is not, you must provide a counterexample.**

   (a) $S = \mathbb{Q}^+, a * b = \dfrac{a+b}{a^2+b^2}$

   This is a binary operation.

   (b) $S = \mathbb{Z}^+, a * b = a - b$

   This is not a binary operation.

   For a counterexample, take $a = 3, b = 7$.

   Even though $a, b \in \mathbb{Z}^+, a - b = -4 \notin \mathbb{Z}^+$.

3. Determine if the given binary operation on the set S is commutative and if it is associative. **If you determine that it is not commutative and/or associative, you must provide a counterexample.**

   (a) $S = \mathbb{Z}, a * b = b$

   This is not commutative.

   For example, $2 * 3 \neq 3 * 2$.

   This is associative, since for all $a, b, c \in \mathbb{Z}$, we have

   $a * (b * c) = a * c = c$ and $(a * b) * c = c$.

   (b) $S = \mathbb{Z}, a * b = a^2 + b^2$

   This is commutative, since for all $a, b \in \mathbb{Z}$, we have

   $a^2 + b^2 = b^2 + a^2$.

   This is not associative.

   For example, $0 * (1 * 1) = 0 * \left(1^2 + 1^2\right) = 0 * 2 = 0^2 + 2^2 = 4$,

   but, $(0 * 1) * 1 = \left(0^2 + 1^2\right) * 1 = 1 * 1 = 1^2 + 1^2 = 2 \neq 4$.