Math 361 Name KEY

Quiz 2

Note: The proofs below are only examples. There are many different ways to prove each statement. You are encouraged to find a style that you feel comfortable with. The only requirement is that the logic is clear and sound.

1. Let $(G, *)$ be a group. Prove: If $x, y, z \in G$ such that $z = x * y$, then $y^{-1} = z^{-1} * x$. (You may use any of the theorems we have proved in class, as long as you explain each step.)

Proof. Premises: $x, y, z \in G$ and $z = x * y$.

Starring on the right by y^{-1} on each side of the equation $z = x * y$ gives us $z * y^{-1} = x * y * y^{-1}$, which simplifies to $z * y^{-1} = x$, since $x * y * y^{-1} = x * e = x$.

Starring each side of the equation $z * y^{-1} = x$ on the left by z^{-1}, we have $z^{-1} * z * y^{-1} = z^{-1} * x$.

Since $z^{-1} * z * y^{-1} = e * y^{-1} = y^{-1}$, we have the following conclusion.

Conclusion: $y^{-1} = z^{-1} * x$. ■

2. Let $(G, *)$ be a group. Prove: $\forall x, y \in G, \left(x * y * x^{-1}\right)^2 = x * y^2 * x^{-1}$. (Here a^3 means $a * a * a$.)

Proof. Premises: None.

Let $x, y \in G$ be arbitrarily chosen.

$$\begin{aligned}\left(x * y * x^{-1}\right)^2 &= \left(x * y * x^{-1}\right) * \left(x * y * x^{-1}\right) \\ &= x * y * \left(x^{-1} * x\right) * y * x^{-1} \\ &= x * y * e * y * x^{-1} \\ &= x * y * y * x^{-1}\end{aligned}$$

Since x, y were arbitrarily chosen, we have the following conclusion.

Conclusion: $\left(x * y * x^{-1}\right)^2 = x * y^2 * x^{-1}, \forall x, y \in G$. ■

3. Determine if each set S forms a group with the stated binary operation $*$. If it is not a group, clearly explain why it is not. If it is a group, state the identity element of the group and state 2 other elements and their inverses.

(a) $S =$ the set of nonnegative even integers, $* =$ ordinary addition.

This is not a group. The identity element in S with $*$ is 0.
The inverse of, for example, 2 would be -2, but $-2 \notin S$.

(b) $S =$ the set of nonnegative real numbers, $* =$ ordinary multiplication.

This is not a group. The identity element in S with $*$ is 1.
Unfortunately, 0 has no inverse. (Every other element in S does, though.)

(c) $S = \mathbb{Z}_8$, $* =$ addition modulo 8 (that is, $\oplus$ in the context of $\mathbb{Z}_8$.)

This is a group. The identity element is 0.
Two elements in the group are 2 and 3.
There inverses are 6 and 5, respectively
(because $2 \oplus 6 = 0$ and $3 \oplus 5 = 0$.)