Math 361 Name KEY
Quiz 6

1. Let $P = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 2 & 1 & 5 \end{pmatrix}$, and $Q = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 2 & 1 & 5 & 4 \end{pmatrix}$.

   (a) Compute $P \circ Q$. (be careful.)

   $$P \circ Q = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 3 & 5 & 1 \end{pmatrix}$$

   (b) Determine if each of these permutations is even or odd.

   $P = (1,3,2,4) = (1,4) \circ (1,2) \circ (1,3)$. So, P is odd.
   $Q = (1,3) \circ (4,5)$. So Q is even.

   (c) Find the order of each of these permutations (as elements of the group S_5).

   $o(P) = 4$ (since it can be written as the 4-cycle $(1,3,2,4)$).
   $o(Q) = 2$ (since it can be written as the composition of two 2-cycles and LCM (2,2) = 2).

2. Determine if each of the following subsets of S_4 are subgroups of S_4. (You do not have to write a formal proof in any case, but be sure to fully explain your reasoning.)

   (a) The set of odd permutations of S_4.

   This is not a subgroup. One reason is the the identity element is even, not odd, so this set does not contain the identity element. Another reason is that the composition of two odd permutations is an even permutation, not an odd one, so this set is not closed under the group operation.

   (b) The set of permutations in S_4 that keep 4 fixed.

   This is a subgroup. You can either check the two parts of the subgtroup theorem, or you can note that this set is the same as S_3, which we already know is a group.