Math 361 Name KEY
Quiz 7

This is a take-home quiz. You may use the class textbook and notes. You are not permitted to work with each other or to get help from the tutors on any of these particular problems.

1. Let $G = S_3$ and $y = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$. Find $Z(y)$ and $\overline{y}$.

$$S_3 = \left\{ \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} \right\}$$

$$\begin{aligned} Z(y) = Z\left(\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \right) &= \text{the set of elements from } S_3 \text{ that commute with } \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \\ &= \left\{ \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} \right\} \end{aligned}$$

$$\begin{aligned} \overline{y} = \overline{\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}} &= \text{the set of all products } a \circ \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \circ a^{-1} \text{ where } a \in S_3 \\ &= \left\{ \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} \right\} \end{aligned}$$

2. Let G be a group, H a subgroup of G, and $K \triangleleft G$. Prove: $H \cap K \triangleleft H$.

Proof: Premises: H is a subgroup of G, $K \triangleleft G$

Let $x \in H \cap K$ and $h \in H$ be PBAC.
Then $x \in H$ and $x \in K$.
Now, since $h \in H$, we have $h^{-1} \in H$ as well.
Thus, $hxh^{-1} \in H$.
Now, since $K \triangleleft G$, and $h \in G$ and $x \in K$, we have $hxh^{-1} \in K$ also.
Thus, $hxh^{-1} \in H \cap K$.

Conclusion: $H \cap K \triangleleft H$.

3. Let $A \triangleleft G$ and $B \triangleleft H$. Show that $A \times B \triangleleft G \times H$. (Hint: a typical element of $A \times B$ may be written as (a, b))

Proof: Premises: $A \triangleleft G$ and $B \triangleleft H$

Let $(a, b) \in A \times B$ and $(g, h) \in G \times H$ be PBAC.
Then $a \in A, b \in B, g \in G$ and $h \in H$.
Now, since $A \triangleleft G$, and $g \in G$ and $a \in A$, we have $gag^{-1} \in A$.
Similarly, since $B \triangleleft H$, and $h \in H$ and $b \in B$, we have $hbh^{-1} \in B$.
Thus, $(gag^{-1}, hbh^{-1}) \in G \times H$
We now note that

$$\begin{aligned} (g, h) * (a, b) * (g, h)^{-1} &= (g, h) * (a, b) * (g^{-1}, h^{-1}) \\ &= (gag^{-1}, hbh^{-1}) \end{aligned}$$

Therefore, $(g, h) * (a, b) * (g, h)^{-1} \in G \times H$.

Conclusion: $A \times B \triangleleft G \times H$.