

This is a take-home quiz. You may use the class textbook and notes. You are not permitted to work with each other or to get help from the tutors on any of these particular problems.

- (a) Clearly explain why  $\langle (2, 1) \rangle \triangleleft \mathbb{Z}_6 \times \mathbb{Z}_3$ .

$\mathbb{Z}_6 \times \mathbb{Z}_3$  is an abelian group. Every subgroup of an abelian group is normal.

- (b) Find all cosets of  $\langle (2, 1) \rangle$  in  $\mathbb{Z}_6 \times \mathbb{Z}_3$ .

The cosets are:

- $\{(0, 0), (2, 1), (4, 2)\}$  – This is just  $\langle (2, 1) \rangle$  itself.
  - $\{(0, 1), (2, 2), (4, 0)\}$  – This can be written as  $\langle (2, 1) \rangle + (0, 1)$
  - $\{(0, 2), (2, 0), (4, 1)\}$  – This can be written as  $\langle (2, 1) \rangle + (0, 2)$
  - $\{(1, 0), (3, 1), (5, 2)\}$  – This can be written as  $\langle (2, 1) \rangle + (1, 0)$
  - $\{(1, 1), (3, 2), (5, 0)\}$  – This can be written as  $\langle (2, 1) \rangle + (1, 1)$
  - $\{(1, 2), (3, 0), (5, 1)\}$  – This can be written as  $\langle (2, 1) \rangle + (1, 2)$
- Notice that every element of  $\mathbb{Z}_6 \times \mathbb{Z}_3$  is in exactly one of the cosets.

- (c) Make a group table for the factor group  $\mathbb{Z}_6 \times \mathbb{Z}_3 / \langle (2, 1) \rangle$ .

Call the 6 cosets listed above  $A, B, C, D, E$ , and  $F$ .

$\otimes$	$A$	$B$	$C$	$D$	$E$	$F$
$A$	$A$	$B$	$C$	$D$	$E$	$F$
$B$	$B$	$C$	$A$	$E$	$F$	$D$
$C$	$C$	$A$	$B$	$F$	$D$	$E$
$D$	$D$	$E$	$F$	$C$	$A$	$B$
$E$	$E$	$F$	$D$	$A$	$B$	$C$
$F$	$F$	$D$	$E$	$B$	$C$	$A$

- (d) Find the order of the following elements in the factor group  $\mathbb{Z}_6 \times \mathbb{Z}_3 / \langle (2, 1) \rangle$ .

i.  $\langle (2, 1) \rangle * (1, 1)$

The order of this element is 6, because you must  $\otimes$  it with itself 6 times to get the identity element  $\langle (2, 1) \rangle$ .

ii.  $\langle (2, 1) \rangle * (4, 2)$

This is the identity element of the factor group. Its order is therefore 1.