1. Use an appropriate trigonometric substitution to rewrite the following integral as a trigonometric integral in terms of the variable θ. Do not evaluate the integral, but do simplify the integrand as much as possible.

$$\int \frac{3x^2}{\sqrt{9x^2+1}}\,dx$$

$$\begin{aligned} 3x &= \tan\theta \\ x &= \frac{1}{3}\tan\theta \\ dx &= \frac{1}{3}(\sec\theta)^2\,d\theta \end{aligned}$$

$$\begin{aligned} \int \frac{3x^2}{\sqrt{9x^2+1}}\,dx &= \int \frac{3\left(\frac{1}{3}\tan\theta\right)^2}{\sqrt{9\left(\frac{1}{3}\tan\theta\right)^2+1}} \cdot \frac{1}{3}(\sec\theta)^2\,d\theta \\ &= \int \frac{3\left(\frac{1}{3}\tan\theta\right)^2}{\sec\theta} \cdot \frac{1}{3}(\sec\theta)^2\,d\theta \\ &= \frac{1}{9}\int (\tan\theta)^2 \sec\theta\ d\theta \end{aligned}$$

2. Evaluate $\int 4x^2e^{5x}\ dx$.

D	A
$4x^2$	e^{5x}
$8x$	$\frac{1}{5}e^{5x}$
8	$\frac{1}{25}e^{5x}$

	D	A	
$+$	$4x^2$	e^{5x}	
$-$	$8x$	$\frac{1}{5}e^{5x}$	
$+\int$	8	$\frac{1}{25}e^{5x}$	dx

$$\begin{aligned} \int 4x^2e^{5x}\ dx &= \frac{4}{5}x^2e^{5x} - \frac{8}{25}xe^{5x} + \frac{8}{25}\int e^{5x} \\ &= \frac{4}{5}x^2e^{5x} - \frac{8}{25}xe^{5x} + \frac{8}{125}e^{5x} + C \end{aligned}$$

3. Evaluate $\int \arctan x\ dx$.

D	A
$\arctan x$	1
$\frac{1}{1+x^2}$	x

	D	A	
$+$	$\arctan x$	1	
$-\int$	$\frac{1}{1+x^2}$	x	dx

$$\int \arctan x\ dx = x\arctan x - \int \frac{x}{1+x^2}dx$$

$$\begin{aligned} u &= 1+x^2 \\ \frac{du}{dx} &= 2x \\ dx &= \frac{du}{2x} \end{aligned}$$

$$\begin{aligned} \int \arctan x \; dx &= x\arctan x - \frac{1}{2}\int \frac{1}{u} du \\ &= x\arctan x - \frac{1}{2}\ln|u| + C \\ &= x\arctan x - \frac{1}{2}\ln\left(1+x^2\right) + C \end{aligned}$$

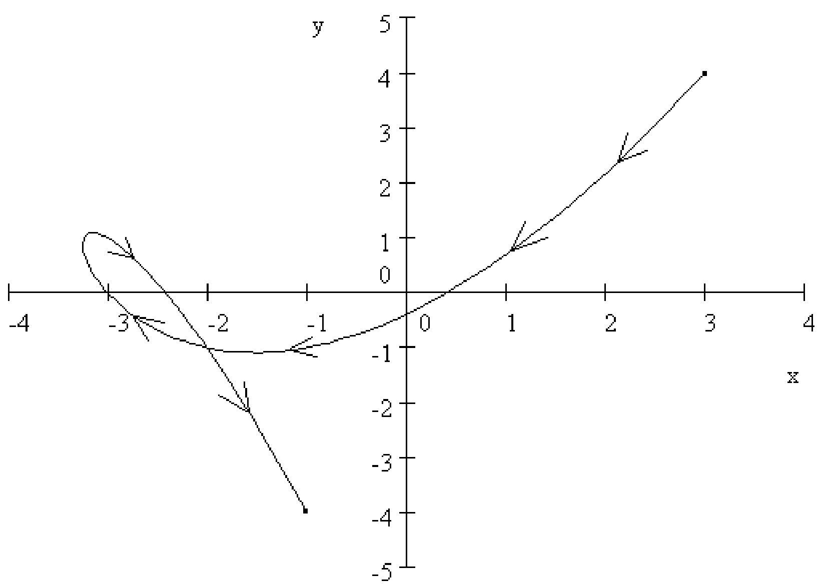
4. Evaluate $\int \dfrac{x}{(x-4)(x-5)}\; dx$.

$$\begin{aligned} \frac{x}{(x-4)(x-5)} &= \frac{A}{x-4} + \frac{B}{x-5} \\ x &= A(x-5) + B(x-4) \end{aligned}$$

$$\begin{aligned} x &= 5: \qquad 5 = B \\ x &= 4: \qquad 4 = -A \qquad \Longrightarrow \qquad A = -4 \end{aligned}$$

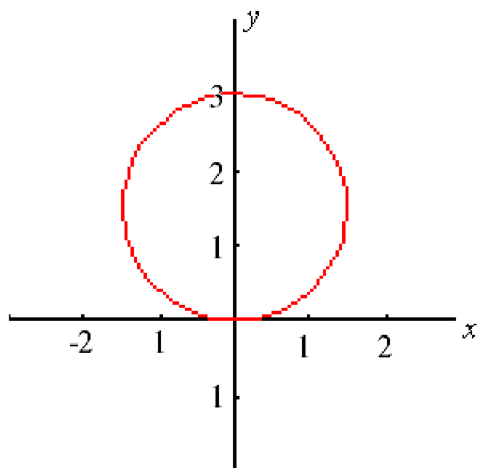
$$\begin{aligned} \int \frac{x}{(x-4)(x-5)}\; dx &= \int \frac{-4}{x-5} + \frac{5}{x-5}\; dx \\ &= -4\ln|x-5| + 5\ln|x-5| + C \end{aligned}$$

5. Let $x = t^2 - t - 3, y = 2t - t^3, -2 \le t \le 2$. Make a careful sketch (on $xy-$axes) of this parametric curve. Indicate with arrows the direction the curve is traced out at t increases.

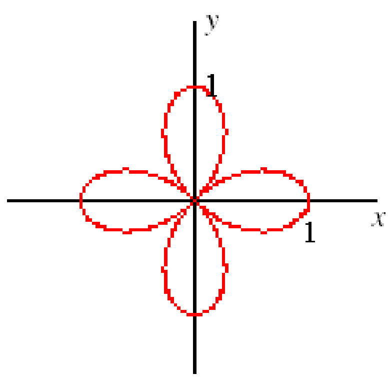


6. Plot each of the polar curves (on $xy-$axes).

(a) $r = 3\sin\theta$



(b) $r = \cos(2\theta)$



7. Find the slope of the line that is:

(a) tangent to the parametric curve defined by $x = 5t^2 + 3t, y = 4t + 2$ at the point $(8, 6)$.

When $x = 8$ and $y = 6$, we have $t = 1$.

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy/dt}{dx/dt} = \frac{10t+3}{4} \\ m &= \left.\frac{dy}{dx}\right|_{t=1} = \frac{13}{4} \end{aligned}$$

(b) tangent to the polar curve $r = \cos\theta$ at the point where $\theta = \frac{\pi}{4}$.

$$\begin{aligned} x &= r\cos\theta = \cos\theta \cdot \cos\theta = (\cos\theta)^2 \\ y &= r\sin\theta = \cos\theta \cdot \sin\theta \end{aligned}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy/d\theta}{dx/d\theta} = \frac{-(\sin\theta)^2 + (\cos\theta)^2}{-2\cos\theta\sin\theta} \\ m &= \left.\frac{dy}{dx}\right|_{\theta=\pi/4} = 0 \end{aligned}$$

8. Short Answer

(a) Write the partial fraction decomposition for the following expression. Do not find the constants.

$$\frac{3x^2+4x+1}{x\left(x-2\right)^3\left(x^2+2\right)^2}$$

$$\frac{3x^2+4x+1}{x\left(x-2\right)^3\left(x^2+2\right)^2}=\frac{A}{x}+\frac{B}{x-2}+\frac{C}{\left(x-2\right)^2}+\frac{D}{\left(x-2\right)^3}+\frac{E}{x^2+2}+\frac{Fx}{x^2+2}+\frac{G}{\left(x^2+2\right)^2}+\frac{Hx}{\left(x^2+2\right)^2}$$

(b) Carefully explain why $\int_{x=-5}^{x=5} \frac{1}{\sqrt[3]{x+4}}\,dx$ is an improper integral.

The integrand is undefined at $x=-4$ and -4 is in the integration range.