Math 152 Quiz 9 Name: KEY

1. Evaluate the following integral or show that it diverges.

$$\int_{x=-1}^{x=1} \frac{1}{x^2}\,dx$$

$$\begin{aligned}
\int_{x=-1}^{x=1} \frac{1}{x^2}dx &= \int_{x=-1}^{x=0} \frac{1}{x^2}dx + \int_{x=0}^{x=1} \frac{1}{x^2}dx \\
&= \lim_{t\to 0^-}\left(\int_{x=-1}^{x=t} \frac{1}{x^2}dx\right) + \lim_{w\to 0^+}\left(\int_{x=w}^{x=1} \frac{1}{x^2}dx\right) \\
&= \lim_{t\to 0^-}\left(-\frac{1}{x}\Big]_{x=-1}^{x=t}\right) + \lim_{w\to 0^+}\left(-\frac{1}{x}\Big]_{x=w}^{x=1}\right) \\
&= \lim_{t\to 0^-}\left(-\frac{1}{t}-1\right) + \lim_{w\to 0^+}\left(-1+\frac{1}{w}\right)
\end{aligned}$$

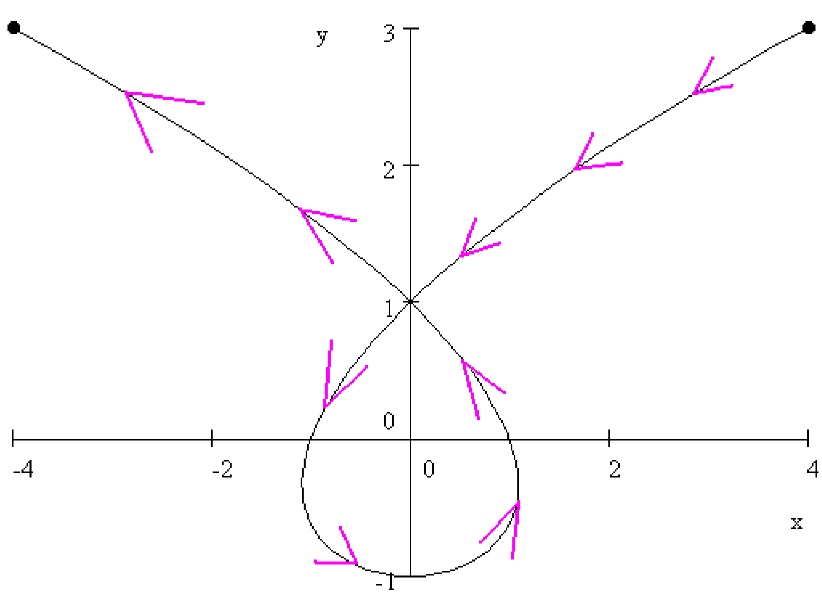
Since at least one of these limits (both of them actually) does not exist, the integral diverges.

2. Consider the curve defined by the following parametric equations.

$$x = 2t - t^3, y = t^2 - 1, -2 \le t \le 2$$

(a) Make a careful sketch of this curve (on the $xy-$axes), by plotting points. Indicate with arrows the direction traced out on the curve as t increases.

t	x	y
-2	4	3
-1	-1	0
0	0	-1
1	1	0
2	-4	3



(b) Find the equation of the line tangent to this curve at the point where $t = 1$.

$$\begin{aligned}
\text{When } t &= 1, \text{ the point on the graph is } (1,0). \\
\text{Now we need the slope there} &: \\
\frac{dy}{dx} &= \frac{dy/dt}{dx/dt} = \frac{2t}{2-3t^2} \\
m &= \left.\frac{dy}{dx}\right|_{t=1} = \frac{2}{-1} = -2 \\
\text{So the tangent line is} &: \\
y - 0 &= (-2)(x-1) \\
y &= -2x + 2
\end{aligned}$$