1. Find the equation of the line tangent to the given function at the indicated point.

   (a) $f(x) = \dfrac{x^3 - 2}{3x}$ at the point where $x = 1$.

   Solution: $f'(x) = \dfrac{3x \cdot (3x^2) - (x^3 - 2) \cdot 3}{(3x)^2} = \dfrac{6x^3 + 6}{9x^2} = \dfrac{2x^3 + 2}{3x^2}$. $f'(1) = \dfrac{4}{3}$. Also, $f(1) = -\dfrac{1}{3}$.
   Therefore, the slope of the line is $\dfrac{4}{3}$, and a point on it is $(1, -\dfrac{1}{3})$.
   The equation of the line is $y - \left(-\dfrac{1}{3}\right) = \dfrac{4}{3}(x - 1)$, which simplifies to $y = \dfrac{4}{3}x - \dfrac{5}{3}$.

   (b) $h(x) = \cos(x^2)$ at the point where $x = \sqrt{\frac{\pi}{3}}$.

   Solution: $h'(x) = -2x\sin(x^2)$. $h'(\sqrt{\frac{\pi}{3}}) = -2\sqrt{\dfrac{\pi}{3}}\dfrac{\sqrt{3}}{2} = -\sqrt{\pi}$. Also, $h(\sqrt{\frac{\pi}{3}}) = \frac{1}{2}$.
   Therefore, the slope of the line is $-\sqrt{\pi}$, and point is $(\sqrt{\frac{\pi}{6}}, \frac{1}{2})$.
   The equation of the line is $y - \frac{1}{2} = -\sqrt{\pi}(x - \sqrt{\frac{\pi}{3}})$, which simplifies to $y = -\sqrt{\pi}x + \dfrac{\pi}{\sqrt{3}} + \dfrac{1}{2}$.

2. Evaluate

   (a) $\displaystyle\int_{x=0}^{x=1} \frac{1}{x^2 - 5x + 6} dx$

   First we note that : $\displaystyle\int_{x=0}^{x=1} \frac{1}{x^2 - 5x + 6} dx = \int_{x=0}^{x=1} \frac{1}{(x-3)(x-2)} dx$

$$\begin{aligned}
\frac{1}{(x-3)(x-2)} &= \frac{A}{x-3} + \frac{B}{x-2} \\
1 &= A(x-2) + B(x-3) \\
x = 2: &\quad 1 = 0 + B(-1) \implies B = -1 \\
x = 3: &\quad 1 = A(1) + 0 \implies A = 1
\end{aligned}$$

$$\begin{aligned}
\int_{x=0}^{x=1} \frac{1}{(x-3)(x-2)} dx &= \int_{x=0}^{x=1} \frac{1}{x-3} + \frac{-1}{x-2} dx \\
&= \ln|x-3| - \ln|x-2| \Big]_{x=0}^{x=1} \\
&= (\ln 2 + \ln 1) - (\ln 3 - \ln 2) \\
&= 2\ln 2 - \ln 3 \\
&= \ln 4 - \ln 3 \\
&= \ln \frac{4}{3}
\end{aligned}$$

   (b) $\displaystyle\int_{x=\pi/6}^{x=\pi/3} \sin x \cos x \; dx$

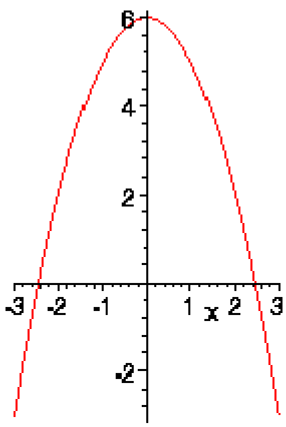
   Note: There are several ways to evaluate this integral. Here is just one of them.

$$\begin{aligned}
u &= \sin x \\
\frac{du}{dx} &= \cos x \\
dx &= \frac{du}{\cos x}
\end{aligned}$$

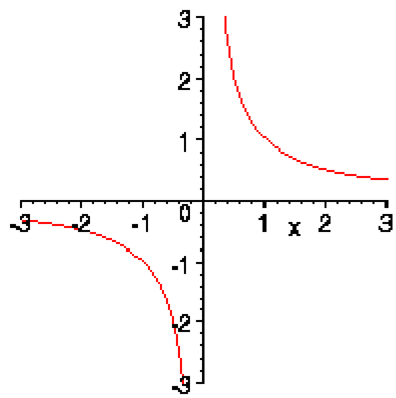
$$\begin{aligned}
\int_{x=\pi/6}^{x=\pi/3} \sin x \cos x \, dx &= \int_{u=1/2}^{u=\sqrt{3}/2} u \, du \\
&= \frac{1}{2}u^2 \Big]_{u=1/2}^{u=\sqrt{3}/2} \\
&= \frac{3}{8} - \frac{1}{8} \\
&= \frac{1}{4}
\end{aligned}$$

3. Sketch each curve on a set of $xy-$ axes.

(a) $f(x) = 6 - x^2$.




(b) $xy = 1$.

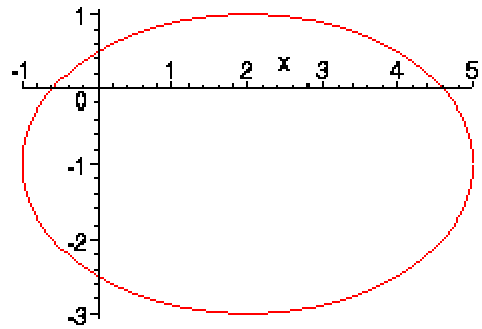



(c) $4x^2 + 9y^2 - 16x + 18y = 11$

This is the equation of a conic section. It will be easier to graph if we complete the squares first.

$$\begin{aligned}
4x^2 + 9y^2 - 16x + 18y &= 11 \\
4x^2 - 16x + 9y^2 + 18y &= 11 \\
4(x^2 - 4x) + 9(y^2 + 2y) &= 11 \\
4(x^2 - 4x + 4 - 4) + 9(y^2 + 2y + 1 - 1) &= 11 \\
4(x^2 - 4x + 4) - 16 + 9(y^2 + 2y + 1) - 9 &= 11 \\
4(x^2 - 4x + 4) + 9(y^2 + 2y + 1) &= 36 \\
4(x-2)^2 + 9(y+1)^2 &= 36 \\
\frac{(x-2)^2}{9} + \frac{(y+1)^2}{4} &= 1
\end{aligned}$$

So, this is an ellipse centered at $(2, -1)$.

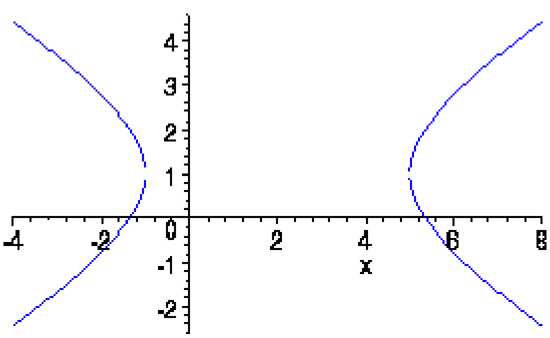



(d) $4x^2 - 9y^2 - 16x - 18y = 29$.

Again, we complete the square.

$$\begin{aligned}
4x^2 - 9y^2 - 16x + 18y &= 29 \\
4x^2 - 16x - 9y^2 + 18y &= 29 \\
4(x^2 - 4x) - 9(y^2 - 2y) &= 29 \\
4(x^2 - 4x + 4 - 4) - 9(y^2 - 2y + 1 - 1) &= 29 \\
4(x^2 - 4x + 4) - 16 - 9(y^2 - 2y + 1) + 9 &= 29 \\
4(x^2 - 4x + 4) - 9(y^2 - 2y + 1) &= 36 \\
4(x-2)^2 - 9(y-1)^2 &= 36 \\
\frac{(x-2)^2}{9} - \frac{(y-1)^2}{4} &= 1
\end{aligned}$$

So this is a hyperbola centered at $(2, 1)$.

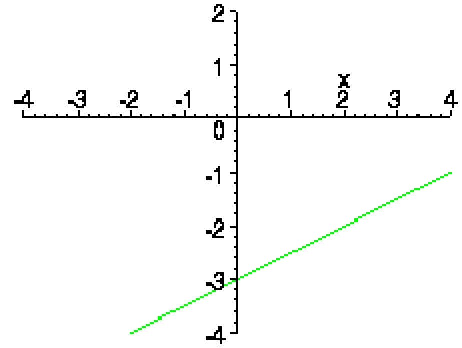



(e) The parametric curve described by $x = 2t + 4, y = t - 1$.

Let's eliminate the parameter t.

$$\begin{aligned}
x &= 2t + 4 \\
t &= \frac{x-4}{2} \\
y &= \left(\frac{x-4}{2}\right) - 1 \\
y &= \frac{1}{2}x - 3
\end{aligned}$$

So this is a straight line. Because there is no constraint on the parameter t, this is the complete line, not just a segment or ray.

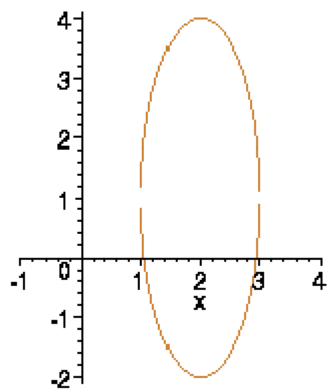



(f) The parametric curve described by $x = 2 + \cos t, y = 1 + 3\sin t$.

Again, we eliminate the parameter.

$$\begin{aligned}
(\cos t)^2 + (\sin t)^2 &= 1 \\
(x-2)^2 + \left(\frac{y-1}{3}\right)^2 &= 1 \\
(x-2)^2 + \frac{(y-1)^2}{9} &= 1
\end{aligned}$$

So this is an ellipse centered at $(2, 1)$.




4. Find the area trapped between the curves $y = 3x^2 + 2, x = -1, x = 2$, and $y = 0$.

   The area is given by the definite integral $\int_{x=-1}^{x=2} 3x^2 + 2 \ dx$

$$\begin{aligned}
\int_{x=-1}^{x=2} 3x^2 + 2 \ dx &= x^3 + 2x]_{x=-1}^{x=2} \\
&= 10 - (-3) \\
&= 13 \text{ square units}
\end{aligned}$$

5. Find the shortest distance from the point $(2, 3)$ to the line $y = 2x$. (You should do this using calculus, but if you want to confirm your answer using plain old algebra, go for it.)

   Suppose $(x, y)$ is a point on the line. Then the distance from that point to $(2, 3)$ is given by the following formula: $d = \sqrt{(x-2)^2 + (y-3)^2}$.

   Because the point is on the line, $y = 2x$, the distance can be written as $d = \sqrt{(x-2)^2 + (2x-3)^2}$. Now, we make things easier by realizing that if we find the x-value that minimizes the distance, we will also have the value that minimizes the square of the distance. The square of the distance is given by $d^2 = (x-2)^2 + (2x-3)^2$. This is the function we want to minimize. Recall that we do this by setting its derivative equal to 0.

$$\begin{aligned}
f(x) &= (x-2)^2 + (2x-3)^2 \\
f(x) &= x^2 - 4x + 4 + 4x^2 - 12x + 9 \\
&= 5x^2 - 16x + 13 \\
f'(x) &= 10x - 16
\end{aligned}$$

Setting $10x - 16 = 0$, gives $x = 8/5$. Since the function is a parabola that opens upward, this x-value must be a minimum point, not a maximum point. The distance that corresponds to this x-value is $\sqrt{(\frac{8}{5} - 2)^2 + (2(\frac{8}{5}) - 3)^2} = \sqrt{(\frac{-2}{5})^2 + (\frac{1}{5})^2} = \sqrt{\frac{4}{25} + \frac{1}{25}} = \sqrt{\frac{1}{5}}$ units.