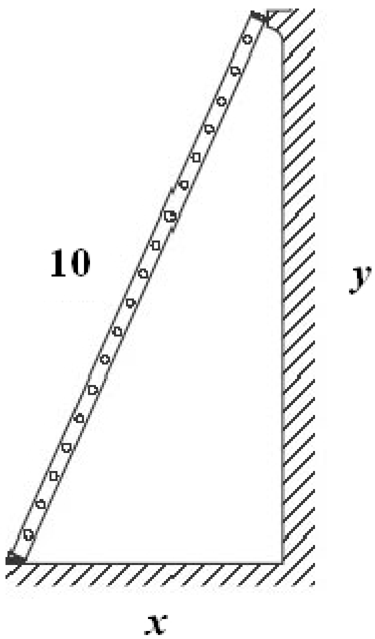
Math 151 Quiz #5 Name: KEY

Show all work. Your answers must be fully justified.

1. A 10 foot ladder is resting against a wall. The ladder begins to slip so that the foot of the ladder begins sliding along the floor away from the wall at a speed of 1 foot per second. How fast is the top of the ladder sliding down the wall at the instant that the foot of the ladder is 6 feet from the wall?



(Picture from the website of The Open University)

Vars.	Rates
$x = 6$	$\frac{dx}{dt} = 1$
$y = 8$	$\frac{dy}{dt} = ?$

$$x^2 + y^2 = 100$$
$$y = \sqrt{100 - 6^2} = 8$$

$$\frac{d}{dt}(x^2 + y^2) = \frac{d}{dt}(100)$$
$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$
$$12 + 16\frac{dy}{dt} = 0$$
$$\frac{dy}{dt} = -\frac{3}{4} \text{ feet/second}$$

2. Let a curve be implicitly defined by $x^2y^2 + y^3 = 6x - 8$.

(a) Find $\frac{dy}{dx}$.

$$\begin{aligned} \frac{d}{dx}\left(x^2y^2 + y^3\right) &= \frac{d}{dx}(6x - 8) \\ 2xy^2 + 2x^2y\frac{dy}{dx} + 3y^2\frac{dy}{dx} &= 6 \\ \frac{dy}{dx} &= \frac{6 - 2xy^2}{2x^2y + 3y^2} \end{aligned}$$

(b) Find the equation of the tangent line to the curve at the point where $x = 0$.

$$\begin{aligned} y^3 &= -8 \\ y &= -2 \end{aligned}$$

$$\left.\frac{dy}{dx}\right|_{x=0, y=-2} = \frac{6}{12} = \frac{1}{2}$$

$$y + 2 = \frac{1}{2}(x - 0)$$

(c) Find $\frac{d^2y}{dx^2}$. (You do not need to simplify and you may leave $\frac{dy}{dx}$ in your answer.)

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{6-2xy^2}{2x^2y+3y^2}\right)$$

$$\frac{d^2y}{dx^2} = \frac{(2x^2y+3y^2)\left(-2y^2-4xy\frac{dy}{dx}\right)-(6-2xy^2)\left(4xy+2x^2\frac{dy}{dx}+6y\frac{dy}{dx}\right)}{(2x^2y+3y^2)^2}$$