

Show all work. Your answers must be fully justified.

1. Evaluate the integral:

$$(a) \int_{x=1}^{x=2} (6x^2 - x + 1) dx$$

$$\begin{aligned} \left(2x^3 - \frac{1}{2}x^2 + x \right) \Big|_{x=1}^{x=2} &= (16 - 2 + 2) - \left(2 - \frac{1}{2} + 1 \right) \\ &= 16 - \frac{5}{2} \\ &= \frac{27}{2} \end{aligned}$$

$$(b) \int \left(\frac{8}{x^3} - \sqrt[3]{x} \right) dx$$

$$\begin{aligned} \int (8x^{-3} - x^{1/3}) dx &= -4x^{-2} - \frac{3}{4}x^{4/3} + C \\ &= -\frac{4}{x^2} - \frac{3}{4}(\sqrt[3]{x})^4 + C \end{aligned}$$

2. Let $b(t)$ represent the rate of growth of bacteria in a lab specimen, where t is measured in days. What does $\int_{t=0}^{t=7} b(t) dt$ represent?

$\int_{t=0}^{t=7} b(t) dt$ represents the total amount of bacteria that grew in the specimen between the time $t = 0$ and the time $t = 7$ (i.e., during the first week.)

3. Let $h(x) = \int_{t=x^4}^{t=2} \frac{t^3 \sin t}{\sqrt{1+t}} dt$. Find $h'(x)$

$$\begin{aligned} h(x) &= - \int_{t=2}^{t=x^4} \frac{t^3 \sin t}{\sqrt{1+t}} dt \\ h'(x) &= - \frac{(x^4)^3 \sin(x^4)}{\sqrt{1+x^4}} \cdot 4x^3 \\ &= - \frac{4x^{15} \sin(x^4)}{\sqrt{1+x^4}} \end{aligned}$$