Worksheet on Functions and Function Notation

1. Rewrite each sentence using function notation. Assume that $y = f(x)$.

(a) When $x = 3, y = 4$.

$$f(3) = 4$$

(b) If $x = 2, y = 7$.

$$f(2) = 7$$

(c) The point $(7, 9)$ is on the graph of the function.

$$f(7) = 9$$

(d) When 4 is put in the function, 8 comes out of the function.

$$f(4) = 8$$

(e) 3 comes out of the function when -2 is put into it.

$$f(-2) = 3$$

(f) The $x-$values 1 and 3 share the same $y-$value.

$$f(1) = f(3)$$

(g) When $x = a, y = b$.

$$f(a) = b$$

(h) When $x = a, y = a^2$.

$$f(a) = a^2$$

2. Let $f(x) = x^2 - 1$.

(a) Find $f(-3)$.

$$f(-3) = (-3)^2 - 1 = 9 - 1 = 8$$

(b) Find $f(b)$.

$$f(b) = b^2 - 1$$

(c) Find $f(b) + 2$.

$$f(b) + 2 = b^2 - 1 + 2 = b^2 + 1$$

(d) Find $f(b + 2)$.

$$f(b+2) = (b+2)^2 - 1 = b^2 - 4b + 4 - 1 = b^2 - 4b + 3$$

(e) Find $\dfrac{f(2+h) - f(2)}{h}$ and simplify as much as possible.

$$\begin{aligned}
\frac{f(2+h) - f(2)}{h} &= \frac{\left((2+h)^2 - 1\right) - \left(2^2 - 1\right)}{h} \\
&= \frac{\left(4 + 4h + h^2 - 1\right) - 3}{h} \\
&= \frac{4h + h^2}{h} \\
&= \frac{h\,(4+h)}{h} \\
&= \frac{\not{h}\,(4+h)}{\not{h}} \\
&= 4 + h
\end{aligned}$$

(f) Find $\dfrac{f(x)-f(2)}{x-2}$ and simplify as much as possible.

$$\begin{aligned}
\frac{f(x)-f(2)}{x-2} &= \frac{\left(x^2-1\right)-\left(2^2-1\right)}{x-2} \\
&= \frac{x^2-4}{x-2} \\
&= \frac{(x+2)(x-2)}{x-2} \\
&= \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}} \\
&= x+2
\end{aligned}$$

3. Let $f(x) = \dfrac{1}{x+1}$.

(a) Find $f(-3)$.

$$f(-3) = \frac{1}{-3+1} = -\frac{1}{2}$$

(b) Find $f(b)$.

$$f(b) = \frac{1}{b+1}$$

(c) Find $f(b)+2$.

$$f(b)+2 = \frac{1}{b+1}+2$$

(d) Find $f(b+2)$.

$$f(b+2) = \frac{1}{(b+2)+1} = \frac{1}{b+3}$$

(e) Find $\dfrac{f(2+h)-f(2)}{h}$ and simplify as much as possible.

$$\begin{aligned}
\frac{f(2+h)-f(2)}{h} &= \frac{\frac{1}{(2+h)+1}-\frac{1}{2+1}}{h} \\
&= \frac{\frac{1}{h+3}-\frac{1}{3}}{h} \\
&= \frac{\frac{3}{3(h+3)}-\frac{h+3}{3(h+3)}}{h} \\
&= \frac{\left(\frac{3-(h+3)}{3(h+3)}\right)}{h} \\
&= \frac{\left(\frac{-h}{3(h+3)}\right)}{h} \\
&= \frac{-h}{3(h+3)}\cdot\frac{1}{h} \\
&= \frac{-\cancel{h}}{3(h+3)}\cdot\frac{1}{\cancel{h}} \\
&= -\frac{1}{3(h+3)} \\
&= -\frac{1}{3h+9}
\end{aligned}$$

(f) Find $\dfrac{f(x)-f(2)}{x-2}$ and simplify as much as possible.

$$
\begin{aligned}
\frac{f(x)-f(2)}{x-2} &= \frac{\dfrac{1}{x+1}-\dfrac{1}{2+1}}{x-2} \\
&= \frac{\dfrac{1}{x+1}-\dfrac{1}{3}}{x-2} \\
&= \frac{\dfrac{3}{3(x+1)}-\dfrac{x+1}{3(x+1)}}{x-2} \\
&= \frac{\left(\dfrac{3-x-1}{3(x+1)}\right)}{x-2} \\
&= \frac{\left(\dfrac{2-x}{3(x+1)}\right)}{x-2} \\
&= \frac{2-x}{3(x+1)}\cdot\frac{1}{x-2} \\
&= \frac{-(x-2)}{3(x+1)}\cdot\frac{1}{x-2} \\
&= \frac{-\left(\cancel{(x-2)}\right)}{3(x+1)}\cdot\frac{1}{\cancel{(x-2)}} \\
&= -\frac{1}{3(x+1)} \\
\backslash \quad &= -\frac{1}{3x+3}
\end{aligned}
$$

4. Let $f(x)=x^2-2x$.

(a) Find all real values of x such that $f(x)=15$.

$$
\begin{aligned}
f(x) &= 15 \\
x^2-2x &= 15 \\
x^2-2x-15 &= 0 \\
(x-5)(x+3) &= 0 \\
x-5 &= 0 \text{ OR } x+3=0 \\
x &= 5 \text{ OR } x=-3
\end{aligned}
$$

(b) Find all real values of x such that $f(x)=0$.

$$
\begin{aligned}
f(x) &= 0 \\
x^2-2x &= 0 \\
x(x-2) &= 0 \\
x &= 0 \text{ OR } x=2
\end{aligned}
$$

(c) Find all real values of x such that $f(x) = 1$.

$$\begin{aligned} f(x) &= 1 \\ x^2 - 2x &= 1 \\ x^2 - 2x - 1 &= 0 \\ x &= \frac{2 \pm \sqrt{4+4}}{2} \quad \text{(from Quadratic Formula)} \\ x &= \frac{2 \pm 2\sqrt{2}}{2} \\ x &= \frac{\not{2} \pm \not{2}\sqrt{2}}{\not{2}} \\ x &= 1 + \sqrt{2} \end{aligned}$$

(d) Find all real values of x such that $f(x) = -15$.

$$\begin{aligned} f(x) &= -15 \\ x^2 - 2x &= -15 \\ x^2 - 2x + 15 &= 0 \\ x &= \frac{2 \pm \sqrt{-56}}{2} \\ & \text{No real solutions} \end{aligned}$$

(e) If $g(x) = x + 4$, find all real values of x such that $f(x) = g(x)$.

$$\begin{aligned} f(x) &= g(x) \\ x^2 - 2x &= x + 4 \\ x^2 - 3x - 4 &= 0 \\ (x-4)(x+1) &= 0 \\ x &= 4 \ \text{ OR } \ x = -1 \end{aligned}$$

5. Find the domain of each of the following functions:

(a) $f(x) = \dfrac{2x}{x-4}$

$(-\infty, 4) \cup (4, \infty)$
That is, the domain includes all real numbers except 4.

(b) $f(x) = x^6 - 3x^2 + 9$

$(-\infty, \infty)$
The domain includes all real numbers.

(c) $f(x) = \sqrt{x-3}$

$[3, \infty)$
The domain includes all real numbers greater than or equal to 3.

(d) $f(x) = \dfrac{x-4}{\sqrt{x-3}}$

$(3, \infty)$
The domain includes all real numbers greater than 3.

(e) $f(x) = \sqrt{5-x}$

$(\infty, 5]$
The domain includes all real numbers less than or equal to 5.

(f) $f(x) = \sqrt[3]{5-x}$

$(-\infty, \infty)$
The domain includes all real numbers.

6. Let $f(x) = \dfrac{4}{x^2 - x}, g(x) = \dfrac{2}{x}$.

(a) Find $f \circ g$.

$$f \circ g(x) = f(g(x)) = \frac{4}{\left(\frac{2}{x}\right)^2 - \frac{2}{x}} = \frac{4}{\frac{4}{x^2} - \frac{2}{x}} = \frac{2x^2}{2-x}, \qquad x \neq 0$$

(b) Find $g \circ f$.

$$g \circ f(x) = g(f(x)) = \frac{2}{\left(\frac{4}{x^2 - x}\right)} = \frac{x^2 - x}{2}, \qquad x \neq 0, 1$$

(c) Find $(f \circ g)(2)$

$$(f \circ g)(2) = f(g(2)) = f(1) \text{ is undefined.}$$

(d) Find the domain of $f \circ g$.

$(-\infty, 0) \cup (0, 2) \cup (2, \infty)$
The domain includes all real numbers except 0 and 2.

(e) Find the domain of $g \circ f$.

$(-\infty, 0) \cup (0, 1) \cup (1, \infty)$
The domain includes all real numbers except 0 and 1.