

Show all work. Your answers must be fully justified.

1. A gumball machine has 10 red gumballs, 20 blue gumballs, and 25 yellow gumballs. If two gumballs are dispensed randomly (without replacement) from the machine, what is the probability that:

(a) the first is red and the second is yellow?

$$P(\text{red then yellow}) = \frac{10}{55} \cdot \frac{25}{54}$$

(b) both are yellow?

$$P(\text{both yellow}) = \frac{25}{55} \cdot \frac{24}{54}$$

(c) neither is yellow?

$$P(\text{both non-yellow}) = \frac{30}{55} \cdot \frac{29}{54}$$

2. For the following probability distribution (showing all possible outcomes):

$x$	$P(x)$
40	0.1
50	0.3
60	0.2
70	0.4

- (a) Complete the probabilities so that the distribution makes sense and so that outcome 70 is twice as likely as the outcome 60.
- (b) Find the expected value of the distribution.

$$\begin{aligned} E(X) &= 40 * 0.1 + 50 * 0.3 + 60 * 0.2 + 70 * 0.4 \\ &= 4 + 15 + 12 + 28 \\ &= 59 \end{aligned}$$

3. Suppose that based on earlier studies, it has been determined that 80% of finches are carriers of a particular gene. If an ornithologist randomly chooses 15 finches to observe, what is the probability that:

(a) exactly 5 of them have the gene?

$$P(X = 5 | n = 15, p = 0.8) = \frac{15!}{5!10!} (0.8)^5 (0.2)^{10}$$

(b) fewer than 3 of them have the gene?

$$P(X < 3 | n = 15, p = 0.8) = \frac{15!}{0!15!} (0.8)^0 (0.2)^{15} + \frac{15!}{1!14!} (0.8)^1 (0.2)^{14} + \frac{15!}{2!13!} (0.8)^2 (0.2)^{13}$$