

Math 301 Exam 1(a)
March 4, 2009

Name _____

1. Clearly explain why each of the following statements are true. (Points will be given for both accuracy and clarity.)

(a) $-6 \mid 18$.

(b) 24 is composite.

(c) $\{x, y\} \in 2^{\{x, y, z\}}$

2. Show that each of the following statements are false by demonstrating a counterexample.

(a) If $x \in \mathbb{Z}$ and $4x$ is even, then x is even.

(b) An integer $x > 0$ if and only if $x^2 > 0$.

(c) $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, xy = 3$.

3. Prove $\neg(x \vee (\neg y))$ is logically equivalent to $(\neg x) \wedge y$.

4. Let $x \in \mathbb{Z}$. Prove: If $3x$ is even, then $9x + 1$ is odd.

5. Suppose that at a certain college, students are randomly assigned ID codes that consist of 5 digits (0-9) followed by 4 letters (A-Z).

(a) How many different ID codes are possible?

(b) How many have no repeated characters (i.e., digits or letters)?

(c) How many have no digits higher than 7?

6. Let $A = \{x \in \mathbb{Z} : 2x > 9\}$ and $B = \{x \in \mathbb{Z} : x > 1\}$. Prove $A \subseteq B$.

7. (a) Write the following sentence using quantifier notation.

There is an integer that is less than the square of every integer.

(b) Write the negation of the following statement.

$$\forall x \in \mathbb{N}, \exists y \in \mathbb{Z}, x + y = 4.$$

without using the symbols \neg , \nexists , or \forall .