

Math 301 Exam 2(a)
April 8, 2009

Name _____

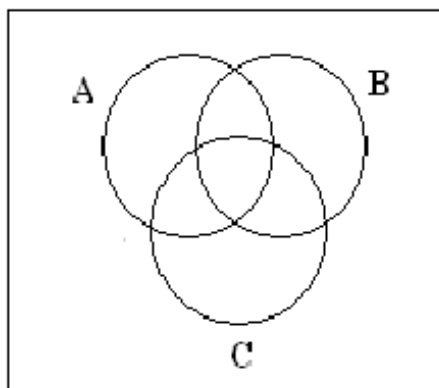
1. (a) Let $A = \{1, 3, 5\}$, $B = \{1, 2, 3, 4\}$. Find each of the following.

i. $A \cap B$

ii. $B - A$

iii. $A \times B$

(b) Let A, B , and C be sets. Shade the Venn Diagram below to represent $A \cup (B - C)$.



2. Let A, B, C , and D be sets. Prove: If $B \subseteq D$ and $C \subseteq D$ then $A \times (B \cup C) \subseteq A \times D$.

3. (a) Let $A = \{1, 2, 3, 4, 5, 6\}$ and R be the relation on A defined by aR_b if $a - b \geq 4$. Write R by listing its elements between curly braces.

(b) Let $A = \{3, 4, 5, 6\}$ and $B = \{4, 6, 8, 10, 12\}$ and R be the relation from A to B defined by aR_b if $a \mid b$. Find $|R|$.

4. (a) Let R be the relation on the set of positive integers given by aR_b if $a^2 = b$. Prove R is antisymmetric.

(b) Let R be the relation on \mathbb{Z} given by aR_b if $a + 1 \mid b$. Provide a counterexample to prove that R is not an equivalence relation.

5. (a) Let R be the equivalence relation on $\{x \in \mathbb{Z} : -20 \leq x \leq 20\}$ defined by aRb if $a \equiv b \pmod{4}$. Write $[7]$ by listing its elements between curly braces.

(b) Let A be a set and let R be an equivalence relation on A . Let $a, x, y \in A$.
Prove: If xRy and $x \in [a]$ then $y \in [a]$.

6. (a) Write the contrapositive of the following statement:

If $x < y$ and $z \in T$, then $a + b = c$.

(b) Let $n \in \mathbb{Z}$. Prove by contradiction: If $3 \nmid n$ then $6 \nmid n$.

7. Provide an example that satisfies the requirements.

(a) Nonempty sets A and B such that $A\Delta B = A \cup B$.

(b) A nonempty set A and a relation R on A such that R is neither reflexive nor irreflexive.
You may represent A and R in any of the ways used in class.

(c) A nonempty set A and a relation R on A such that R is symmetric but not transitive.
You may represent A and R in any of the ways used in class.