1. (a) Let $A = \{2,3,4\}, B = \{3,4,5,6\}$. Find each of the following.

i. $A \cap B$

$$\{3,4\}$$

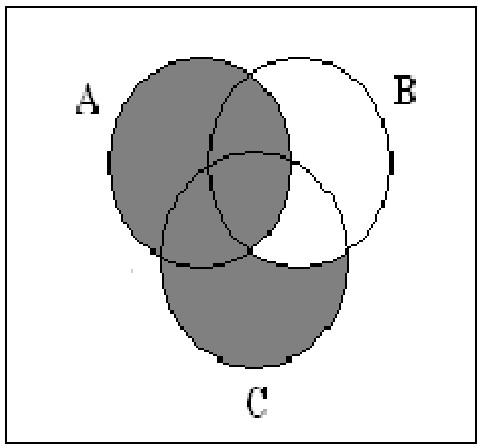
ii. $B - A$

$$\{5,6\}$$

iii. $A \times B$

$$\{(2,3),(2,4),(2,5),(2,6),(3,3),(3,4),(3,5),(3,6),(4,3),(4,4),(4,5),(4,6)\}$$

(b) Let $A, B,$ and C be sets. Shade the Venn Diagram below to represent $A \cup (C - B)$.



2. Let $A, B, C,$ and D be sets. Prove: If $B \subseteq D$ and $C \subseteq D$ then $A \times (B \cup C) \subseteq A \times D$.

Proof: Hyp: $B \subseteq D$ and $C \subseteq D$
Let $x \in A \times (B \cup C)$
Then $x = (y, z)$ for some $y \in A$ and $z \in B \cup C$.
Since $z \in B \cup C$, we have $z \in B$ or $z \in C$.
If $z \in B$ then $z \in D$, since $B \subseteq D$.
If $z \in C$ then $z \in D$, since $C \subseteq D$.
In either case, $z \in D$.
Thus $x \in A \times D$.
Conc: $A \times (B \cup C) \subseteq A \times D$.

3. (a) Let $A = \{1, 2, 3, 4, 5, 6\}$ and R be the relation on A defined by ${}_aR_b$ if $a - b \geq 4$. Write R by listing its elements between curly braces.

$$\{(6,1), (6,2), (5,1)\}$$

(b) Let $A = \{3, 4, 5, 6\}$ and $B = \{6, 8, 10, 12\}$ and R be the relation from A to B defined by ${}_aR_b$ if $a \mid b$. Find $|R|$

$$\begin{aligned} R &= \{(3,6), (3,12), (4,8), (4,12), (5,10), (6,6), (6,12)\} \\ \text{So } |R| &= 7. \end{aligned}$$

4. (a) Let R be the relation on the set of positive integers given by ${}_aR_b$ if $a^2 = b$.
Prove R is antisymmetric.

Proof: Let $x, y \in \mathbb{Z}^+$ such that ${}_xR_y$ and ${}_yR_x$.
Then $x^2 = y$ and $y^2 = x$
So $x^4 = x$ and $y^4 = y$.
$x = 1$ and $y = 1$.
$x = y$.
Conc: R is antisymmetric.

(b) Let R be the relation on $\mathbb{Z}$ given by ${}_aR_b$ if $a + 1 \mid b$.
Provide a counterexample to prove that R is not an equivalence relation.

Since R is not reflexive, symmetric, nor transitive, you can provide a counterexample to any of these properties.
Let $a = 3$. Then $3 + 1 \nmid 3$, so 3 is not related to itself. Thus R is not reflexive.

5. (a) Let R be the equivalence relation on $\{x \in \mathbb{Z} : -20 \leq x \leq 20\}$ defined by ${}_aR_b$ if $a \equiv b \pmod 4$. Write $[5]$ by listing its elements between curly braces.

$$\{-19, -15, -11, -7, -3, 1, 5, 9, 11, 13, 17\}$$

(b) Let A be a set and let R be an equivalence relation on A. Let $a, x, y \in A$.
Prove: If ${}_xR_y$ and $x \in [a]$ then $y \in [a]$.

Proof: Hyp: ${}_xR_y$ and $x \in [a]$.
Since $x \in [a]$, ${}_xR_a$.
Because R is symmetric and ${}_xR_y$, ${}_yR_x$.
Since R is transitive and ${}_yR_x$ and ${}_xR_a$, ${}_yR_a$.
Conc: $x \in [a]$.

6. (a) Write the contrapositive of the following statement:

If $x < y$ and $z \in T$, then $a + b = c$.

If $a + b \neq c$ then $x \geq y$ or $z \notin T$.

(b) Let $n \in \mathbb{Z}$. Prove by contradiction: If $3 \nmid n$ then $6 \nmid n$.

Proof: Hyp: $3 \nmid n$.
Suppose, FSOC, that $6 \mid n$.
Then $n = 6m$ for some $m \in \mathbb{Z}$.
So $n = 3(2m)$ and $2m \in \mathbb{Z}$.
Thus $3 \mid n$.
This contradicts our hypothesis.
Conc: $6 \nmid n$.

7. Provide an example that satisfies the requirements.

(a) Nonempty sets A and B such that $A \triangle B = A \cup B$.

$$A = \{1, 2, 3\}, B = \{4, 5, 6\}$$

(b) A nonempty set A and a relation R on A such that R is neither reflexive nor irreflexive. You may represent A and R in any of the ways used in class.

$$A = \{1, 2, 3\}, R = \{(1, 1)\}$$

(c) A nonempty set A and a relation R on A such that R is symmetric but not transitive. You may represent A and R in any of the ways used in class.

$$A = \{1, 2, 3\}, R = \{(1, 2), (2, 1)\}$$