

1. (a) Let $A = \{1, 3, 5\}$, $B = \{1, 2, 3, 4\}$. Find each of the following.

i. $A \cap B$

$\{1, 3\}$

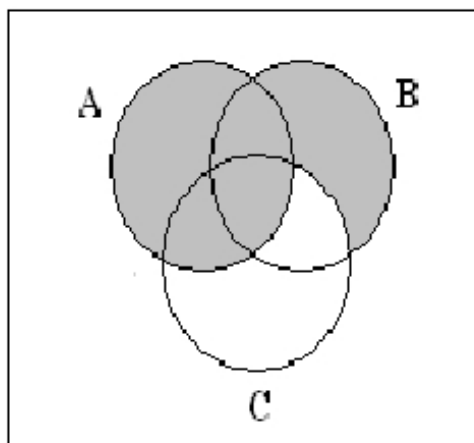
ii. $B - A$

$\{2, 4\}$

iii. $A \times B$

$\{(1, 1), (1, 2), (1, 3), (1, 4), (3, 1), (3, 2), (3, 3), (3, 4), (5, 1), (5, 2), (5, 3), (5, 4)\}$

(b) Let A, B , and C be sets. Shade the Venn Diagram below to represent $A \cup (B - C)$.



2. Let A, B, C , and D be sets. Prove: If $B \subseteq D$ and $C \subseteq D$ then $A \times (B \cup C) \subseteq A \times D$.

Proof: Hyp: $B \subseteq D$ and $C \subseteq D$

Let $x \in A \times (B \cup C)$

Then $x = (y, z)$ for some $y \in A$ and $z \in B \cup C$.

Since $z \in B \cup C$, we have $z \in B$ or $z \in C$.

If $z \in B$ then $z \in D$, since $B \subseteq D$.

If $z \in C$ then $z \in D$, since $C \subseteq D$.

In either case, $z \in D$.

Thus $x \in A \times D$.

Conc: $A \times (B \cup C) \subseteq A \times D$.

3. (a) Let $A = \{1, 2, 3, 4, 5, 6\}$ and R be the relation on A defined by aR_b if $a - b \geq 4$. Write R by listing its elements between curly braces.

$$\{(6, 1), (6, 2), (5, 1)\}$$

- (b) Let $A = \{3, 4, 5, 6\}$ and $B = \{4, 6, 8, 10, 12\}$ and R be the relation from A to B defined by aR_b if $a \mid b$. Find $|R|$.

$$R = \{(3, 6), (3, 12), (4, 4), (4, 8), (4, 12), (5, 10), (6, 6), (6, 12)\}$$

So $|R| = 8$.

4. (a) Let R be the relation on the set of positive integers given by aR_b if $a^2 = b$. Prove R is antisymmetric.

Proof: Let $x, y \in \mathbb{Z}^+$ such that xR_y and yR_x .
Then $x^2 = y$ and $y^2 = x$
So $x^4 = x$ and $y^4 = y$.
 $x = 1$ and $y = 1$.
 $x = y$.
Conc: R is antisymmetric.

- (b) Let R be the relation on \mathbb{Z} given by aR_b if $a + 1 \mid b$.

Provide a counterexample to prove that R is not an equivalence relation.

Since R is not reflexive, symmetric, nor transitive, you can provide a counterexample to any of these properties.

Let $a = 3$. Then $3 + 1 \nmid 3$, so 3 is not related to itself. Thus R is not reflexive.

5. (a) Let R be the equivalence relation on $\{x \in \mathbb{Z} : -20 \leq x \leq 20\}$ defined by aRb if $a \equiv b \pmod{4}$. Write $[7]$ by listing its elements between curly braces.

$$\{-17, -13, -9, -5, -1, 3, 7, 11, 15, 19\}$$

- (b) Let A be a set and let R be an equivalence relation on A . Let $a, x, y \in A$.
Prove: If xRy and $x \in [a]$ then $y \in [a]$.

Proof: Hyp: xRy and $x \in [a]$.

Since $x \in [a]$, xRa .

Because R is symmetric and xRy , yRx .

Since R is transitive and yRx and xRa , yRa .

Conc: $x \in [a]$.

6. (a) Write the contrapositive of the following statement:

If $x < y$ and $z \in T$, then $a + b = c$.

If $a + b \neq c$ then $x \geq y$ or $z \notin T$.

- (b) Let $n \in \mathbb{Z}$. Prove by contradiction: If $3 \nmid n$ then $6 \nmid n$.

Proof: Hyp: $3 \nmid n$.

Suppose, FSOC, that $6 \mid n$.

Then $n = 6m$ for some $m \in \mathbb{Z}$.

So $n = 3(2m)$ and $2m \in \mathbb{Z}$.

Thus $3 \mid n$.

This contradicts our hypothesis.

Conc: $6 \nmid n$.

7. Provide an example that satisfies the requirements.

(a) Nonempty sets A and B such that $A\Delta B = A \cup B$.

$$A = \{1, 2, 3\}, B = \{4, 5, 6\}$$

(b) A set A and a relation R on A such that R is neither reflexive nor irreflexive. You may represent A and R either as sets, or using a diagram, as in class.

$$A = \{1, 2, 3\}, R = \{(1, 1)\}$$

(c) A set A and a relation R on A such that R is symmetric but not transitive. You may represent A and R either as sets, or using a diagram, as in class.

$$A = \{1, 2, 3\}, R = \{(1, 2), (2, 1)\}$$