Math 301 Exam 3(a)
May 6, 2009

Name____________________________

No Calculators allowed.
Do not leave any question blank.
Make sure you have 7 numbered questions.

1. Prove by Mathematical Induction: $\forall n \in \mathbb{N}$, $\frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} + \ldots + \frac{1}{(2n+1)(2n+3)} = \frac{n+1}{2n+3}$.

2. Let $f = \{(x, y) : x \in \mathbb{Z}, y \in \mathbb{R}, x - 2y = 4\}$.

   (a) Find $f(10)$.

   (b) Clearly explain why f is not onto $\mathbb{R}$.

3. Let $f : \mathbb{Z}^{\text{odd}} \to \mathbb{Z}$ be given by $f(x) = \frac{x-1}{2}$.

   (a) Prove that f is one-to-one.

   (b) Prove that f is onto.

4. Calculations:

   (a) Find $\gcd(182, 140)$.

   (b) Find x so that x div $6 = 3$ and $x \bmod 6 = 2$.

5. Let $f = \{(3,8), (4,6), (5,7), (6,9)\}$ and $g = \{(2,3), (3,5), (4,5)\}$. Find each of the following:

   (a) $f^{-1}(9)$

   (b) $f \circ g(3)$.

6. Give an example of each of the following:

   (a) A function $f : \mathbb{Z} \to \mathbb{Z}$ that is not one-to-one.

   (b) An integer that is relatively prime to 12.

7. (a) Disprove the following by demonstrating a counterexample: If $a, b, n \in \mathbb{Z}$ such that $a \bmod n = 1$ then $ab \bmod n = 1$.

   (b) Prove: If $a, b, n \in \mathbb{Z}$ such that $a \bmod n = 1$ and $b \bmod n = 1$ then $ab \bmod n = 1$