3 Exercises

3.1. Each of the following statements can be recast in the if-then form. Please rewrite each of the following sentences in the form "If A, then B."

a. The product of an odd integer and an even integer is even.

b. The square of an odd integer is odd.

c. The square of a prime number is not prime.

d. The product of two negative integers is negative. (This, of course, is false.)

The statement "If B, then A" is called the *converse* of the statement "If A, then B."

3.2. It is a common mistake to confuse the following two statements:

a. If A, then B.

b. If B, then A.

Find two conditions A and B such that statement (a) is true but statement (b) is false.

3.3. Consider the two statements

a. If A, then B.

b. (not A) or B.

Under what circumstances are these statements true? When are they false? Explain why these statements are, in essence, identical.

The statement "If (not B), then (not A)" is called the *contrapositive* of the statement "If A, then B."

3.4. Consider the two statements

a. If A, then B.

b. If (not B), then (not A).

Under what circumstances are these statements true? When are they false? Explain why these statements are, in essence, identical.

3.5. Consider the two statements

a. A iff B.

b. (not A) iff (not B).

Under what circumstances are these statements true? Under what circumstances are they false? Explain why these statements are, in essence, identical.

3.6. Consider an equilateral triangle whose side lengths are $a = b = c = 1$. Notice that in this case $a^2 + b^2 \neq c^2$. Explain why this is not a violation of the Pythagorean Theorem.

A side of a spherical triangle is an arc of a great circle of the sphere on which it is drawn.

3.7. Explain how to draw a triangle on the surface of a sphere that has three right angles. Do the legs and hypotenuse of such a right triangle satisfy the condition $a^2 + b^2 = c^2$? Explain why this is not a violation of the Pythagorean Theorem.

3.8. Consider the sentence "A line is the shortest distance between two points." Strictly speaking, this sentence is nonsense.

Find two errors with this sentence and rewrite it properly.

3.9. Consider the following rather grotesque claim: "If you pick a guinea pig up by its tail, then its eyes will pop out." Is this true?

3.10. What are the two plurals of the word *lemma*?