4 Exercises

4.1. Prove that the sum of two odd integers is even.

4.2. Prove that the sum of an odd integer and an even integer is odd.

4.3. Prove that the product of two even integers is even.

4.4. Prove that the product of an even integer and an odd integer is even.

4.5. Prove that the product of two odd integers is odd.

4.6. Suppose a, b, and c are integers. Prove that if $a|b$ and $a|c$, then $a|(b+c)$.

4.7. Suppose a, b, and c are integers. Prove that if $a|b$, then $a|(bc)$.

4.8. Suppose a, b, d, x, and y are integers. Prove that if $d|a$ and $d|b$, then $d|(ax+by)$.

4.9. Suppose a, b, c, and d are integers. Prove that if $a|b$ and $c|d$, then $(ac)|(bd)$.

4.10. Let x be an integer. Prove that x is odd if and only if $x+1$ is even.

4.11. Let x be an integer. Prove that $0|x$ if and only if $x=0$.

4.12. Let a and b be integers. Prove that $a<b$ if and only if $a \leq b-1$.

4.13. Prove that an integer is odd if and only if it is the sum of two consecutive integers.

4.14. Suppose you are asked to prove a statement of the form "If A or B, then C." Explain why you need to prove (a) "If A, then C" and also (b) "If B, then C." Why is it not enough to prove only one of (a) and (b)?

4.15. Suppose you are asked to prove a statement of the form "A iff B." The standard method is to prove both $A \Rightarrow B$ and $B \Rightarrow A$.

Consider the following alternative proof strategy: Prove both $A \Rightarrow B$ and (not A) $\Rightarrow$ (not B). Explain why this would give a valid proof.