5 Exercises

5.1. Disprove: If a and b are integers with $a|b$, then $a \leq b$.

5.2. Disprove: If a and b are nonnegative integers with $a|b$, then $a \leq b$.

Note: A counterexample to this statement would also be a counterexample for the previous problem, but not necessarily vice versa.

5.3. Disprove: If a, b, and c are positive integers with $a|(bc)$, then $a|b$ or $a|c$.

5.4. Disprove: If a, b, and c are positive integers, then $a^{(b^c)} = (a^b)^c$.

5.5. Consider the polynomial n^2+n+41. Calculate the value of this polynomial for $n = 1, 2, 3, \ldots, 10$. Notice that all the numbers you computed are prime.

Disprove: If n is a positive integer, then $n^2 + n + 41$ is prime.

5.6. What does it mean for an if-and-only-if statement to be false? What properties should a counterexample for an if-and-only-if statement have?

5.7. Disprove: An integer x is positive if and only if $x + 1$ is positive.

5.8. Disprove: Two right triangles have the same area if and only if the lengths of their hypotenuses are the same.

5.9. Disprove: A positive integer is composite if and only if it has two different prime factors.